Effect of inclined thick fin on natural convection in a cavity heated from bottom

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Abstract: In this study, natural convection heat transfer in a square cavity with an adiabatic fin mounted on a vertical wall was investigated numerically. Vertical boundaries were adiabatic and horizontal boundaries were isothermal at different constant temperature. Two-dimensional equations of conservation of mass, momentum and energy were solved using finite difference method. Successive under relaxation (SUR) method was used to solve linear algebraic equations. Results were obtained for various geometrical parameters as the thermal conductivity ratio (RK = 0.1, 1.0 and 10), inclination angle of the fin (30° ≤ φ ≤ 150°), thickness of the fin (0.033 ≤ t ≤ 0.2), and Rayleigh numbers (10³ ≤ Ra ≤ 10⁶). Location and length of fin was fixed as h = w = 0.5. Results were presented with streamlines, isotherms, local and mean Nusselt numbers. It was found that Rayleigh number and the fin mounted on the wall had significant effect on natural convection heat transfer and flow field. The thick fin can be used as control parameter of heat and fluid flow.

Keywords: natural convection; square enclosure; thick fin; numerical solutions; finite element method.


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1 Introduction

Natural convection heat transfer in differentially heated and partitioned cavities are encountered in various industrial applications, such as heating and ventilating of living spaces, fire in buildings, solar thermal collector systems, electronic cooling devices, and in storage of radioactive wastes. Many investigators studied the natural convection for two-dimensional closed cavities with different geometries and boundary conditions. Simple rectangular
cavities with differentially heated vertical walls and adiabatic top and bottom walls were examined both theoretically and experimentally for wide ranges of Rayleigh numbers in earlier studies (De Vahl Davis, 1983; Ostrach, 1972; Bejan, 1982; Yang, 1987; Catton, 1962; Khalifa, 2001a, 2001b).

Also, the fin was used to control heat and mass transfer and flow field in cavities (Nansteel and Greif, 1984; Tasnim and Collins, 2004; Shi and Khodadadi, 2003; Bilgen, 2002; Lin and Bejan, 1983). For instance, Zimmerman and Acharya (1987) studied the free convection heat transfer in a partially divided vertical enclosure with conducting end walls. They reported that the strength of the separation bubble increased while the strength of the main flow decreased with increasing baffle conductivity. Except for low Rayleigh numbers, the Nusselt number values decreased with increasing baffle conductivity. Frederick (1989) studied numerically natural convection in an air-filled, differentially heated, inclined square cavity, with a diathermal partition on its cold wall. He determined heat transfer reduction depended on Rayleigh number, partition length and inclination. Chen et al. (1990) investigated experimentally for steady natural convection in a two dimensional, partially divided, rectangular enclosure, in which one of the vertical walls was maintained at different uniform temperatures and the top and bottom walls were insulated. Results showed that there was a recirculation zone in the upper and left quadrant of the enclosure when there was no opening in the partition plate.

Turkoglu and Yucel (1996) analysed numerically natural convection heat transfer in enclosures with conducting multiple partitions and side walls. Side walls were kept at isothermal conditions, while top and bottom walls were insulated. It was observed that for each Rayleigh number, the mean Nusselt number decreased with increasing partition number. Khalifa and Abdullah (1999) investigated experimentally the effect of different types of partitions on the natural convective heat transfer in enclosures. In their study, the enclosure was fitted with a partition midway between two vertical isothermal walls, one of which was heated and the other was cooled. The results indicated that the location of the opening and the aperture height ratio had significant effects on heat transfer. Dagtekin and Oztop (2001) analysed numerically natural convection heat transfer by heated partitions within enclosure. They investigated the effects of position and heights of the partitions on heat transfer and flow field.

Bilgen (2002) studied numerically laminar and turbulent natural convection in enclosures with partial partitions. In his study, Vertical boundaries were isothermal and horizontal boundaries were adiabatic. The results showed that the flow regime was laminar for Rayleigh number up to $10^8$ thereafter turbulent. Shi and Khodadadi (2003) investigated laminar natural convection heat transfer in a differentially heated square cavity. They proposed a correlation among the mean Nusselt, Rayleigh number, fin length and position. Similarly, Tasnim and Collins (2004) studied numerically the natural convection problem in a square cavity with a baffle on the hot wall and effect of baffle height, length and Rayleigh number on heat transfer performance. They concluded that addition of baffle on the hot wall increased the rate of heat transfer by as much as 31.46% compared with a wall without baffle for $Ra=10^4$.

Ben-Nakhi and Chamkha (2006) studied numerically the effect of length and inclination of a thin fin on natural convection in a square enclosure. A transverse temperature gradient was applied on two opposing walls of the enclosure, while the other two walls were adiabatic. They reported that the Rayleigh number and the thin-fin inclination angle and length had significant effects on the average Nusselt number of the heated wall including the fin of the enclosure. Mezrhab et al. (2006) studied numerically the effect of a single and multiple partitions on heat transfer phenomena in an inclined square cavity. The partitions were attached to the cold wall of the cavity. Results showed that a maximum reduction of the heat transfer occurred between $Ra = 6 	imes 10^3$ and $Ra = 2 	imes 10^4$ when the cavity was divided by one diathermal partition. Varol et al. (2007) investigated numerically natural convection in porous triangular enclosures with a solid adiabatic fin attached to the horizontal wall. The temperature of the bottom wall was higher than that of the inclined wall while the vertical wall was insulated. The obtained results indicated that the fin could be used as a control element for heat transfer and fluid flow. Kandaswamy et al. (2008) studied numerically effect of baffle-cavity ratios on buoyancy convection in a cavity with mutually orthogonal heated baffles. It was found that buoyancy force played a key role and overall heat transfer in the cavity was enhanced for higher values of both baffle-cavity ratios.

Famouri and Hooman (2008) studied numerically entropy generation for natural convection by heated partitions in a cavity. They investigated effect of the Rayleigh number, the position of the heated partition and the dimensionless temperature difference on the local and average entropy generation rate. Other studies related with the subject can be found in the following references (Jami et al., 2006; Bhave et al., 2006; Adams et al., 1999; Moukalled and Acharya, 2001; Hung and Shiau, 1988).

The main aim of the present study was to investigate the effects of attached inclined thick fin on a vertical wall. The fin was attached to control heat and fluid flow of buoyancy driven closed to cavity. The problem was analysed for wide range of parameters including Rayleigh numbers, thermal conductivity ratio, thickness of the fin, and inclination angle of the fin.

2 Enclosure geometry

The considered geometry is shown in Figure 1. It is a two dimensional square enclosure with an inclined fin. Location and length of the fin is fixed as $h = w = 0.5$. Thickness of the fin is given with $t$. The cavity is heated from bottom and cooled from top with isothermal heaters. Vertical walls are adiabatic.
The governing equations of natural convection [equations (1) to (3)] are written in streamfunction-vorticity form for laminar regime in two-dimensional form for steady, incompressible, and Newtonian fluid with the Boussinesq approximation. It is assumed that radiation heat exchange is negligible according to other modes of heat transfer and the gravity acts in vertical direction.

The non-dimensional parameters are listed as

\[ \frac{\partial^2 \Omega}{\partial x^2} + \frac{\partial^2 \Omega}{\partial y^2} = \frac{1}{Pr} \left( \frac{\partial^2 \Psi}{\partial x \partial y} \right) - Ra \frac{\partial \theta}{\partial x} \]  

(1)

\[ \frac{\partial^2 \Omega}{\partial x^2} + \frac{\partial^2 \Omega}{\partial y^2} = \frac{1}{Pr} \left( \frac{\partial^2 \Omega}{\partial x \partial y} \right) - Ra \frac{\partial \theta}{\partial x} \]  

(2)

\[ \frac{\partial \Psi}{\partial x} = \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} \]  

(3)

The non-dimensional parameters are listed as

\[ X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad \theta = \frac{T - T_i}{T_e - T_i}, \quad \Psi = \frac{\psi}{\alpha}, \quad \Omega = \frac{\omega L^2}{\nu}, \quad u = \frac{\partial \Psi}{\partial x}, \quad v = \frac{\partial \Omega}{\partial y}, \quad \frac{\partial^2 \Omega}{\partial x^2} + \frac{\partial^2 \Omega}{\partial y^2} = \frac{1}{Pr} \left( \frac{\partial^2 \Omega}{\partial x \partial y} \right) - Ra \frac{\partial \theta}{\partial x} \]  

(4)

Boundary conditions for the considered model are depicted on the physical model as shown in Figure 1. In this model, \( \Psi = 0 \) for all solid boundaries of enclosure and fin.

On the bottom wall,

\[ \theta(X, 0) = 1, \ \Psi(X, 0) = 0 \]  

(5)

On the top wall,

\[ \theta(X, 1) = 0, \ \Psi(X, 1) = 0 \]  

(6)

On the vertical wall,

\[ \frac{\partial \theta(0, Y)}{\partial x} = 0, \ \frac{\partial \theta(1, Y)}{\partial x} = 0, \ \Psi(0, Y) = 0, \ \Psi(1, Y) = 0 \]  

(7)

In the fin,

\[ \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0 \]  

(8)

For the interface between the fin and flow field,

\[ k_f \frac{\partial \theta}{\partial n} = k_i \frac{\partial \theta}{\partial n} \]  

(9)

Local and mean Nusselt numbers are calculated along the bottom wall using equations [10(a)] and [10(b)], respectively.

\[ Nu_x = \left( - \frac{\partial \theta}{\partial Y} \right)_{y=0} \]  

(10a)

\[ Nu = \int_0^1 Nu_x \, dX \]  

(10b)

Central difference method (CDS) was applied to discretise the governing equations (1) to (3). The solution of linear algebraic equations was performed using successive under relaxation (SUR) method. As convergence criteria, \( 10^{-4} \) was
chosen for all dependent variables and value of 0.1 was taken for under-relaxation parameter. The number of grid points was taken as $61 \times 61$ with uniform spaced mesh in both X and Y directions. The numerical algorithm used in this study was tested with the classical natural convection heat transfer problem in a differentially heated square enclosure. A flow chart for solution is shown in Figure 2. The obtained numerical results were compared with those given in different studies (De Vahl Davis, 1983; Ostrach, 1972; Bejan, 1982; Yang, 1987; Catton, 1962; Khalifa, 2001a, 2001b).

3.1 Validation

The accuracy of the numerical scheme is validated by comparing the average Nusselt number for a differentially heated square enclosure in the absence of the fin for various Rayleigh numbers with those reported by Aydin and Yang (2000). These comparisons show excellent agreement, as shown in Table 1. Table 2 presents a list on average Nusselt number from De Vahl Davis (1983) and present study at different Ra numbers. Deviations are also presented in this table. The table clearly indicated that acceptable result is obtained. In addition, further validation is performed by comparing the present results with the results of various cases of the problem of Aydin and Yang (2000), who considered a similar configuration with a non-inclined fin and reasonable agreement is observed as shown in Figure 3.

### Table 1

<table>
<thead>
<tr>
<th>Ra</th>
<th>Aydin and Yang (2000)</th>
<th>Present study</th>
<th>Deviation (%)</th>
</tr>
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<tbody>
<tr>
<td>$10^3$</td>
<td>2.99</td>
<td>2.91</td>
<td>0.027</td>
</tr>
<tr>
<td>$10^4$</td>
<td>3.91</td>
<td>3.63</td>
<td>0.072</td>
</tr>
<tr>
<td>$10^5$</td>
<td>6.31</td>
<td>5.96</td>
<td>0.055</td>
</tr>
<tr>
<td>$10^6$</td>
<td>11.17</td>
<td>10.93</td>
<td>0.022</td>
</tr>
</tbody>
</table>

Figure 3  Comparison of stream and temperature functions with those of Aydin and Yang (2000) (on the right) and present code (on the left), (a) $Ra = 10^3$  (b) $Ra = 10^4$  (c) $Ra = 10^5$  (d) $Ra = 10^6$
4 Results and discussion

A numerical analysis was performed to obtain natural convection heat transfer and fluid flow in a thick fin attached square enclosure. The parameters were Rayleigh number, thermal conductivity ratio, thickness of the fin \( t \), and inclination angle of the fin \( \phi \). Prandtl number was taken as 0.71 for all calculations. In this section, numerical results for the streamline and temperature contours for various values of the fin inclination angle \( \phi \) and thickness of the fin \( t \) are reported. In addition, representative results for the local and mean Nusselt numbers at various conditions are presented and discussed.

Figure 4 shows the effects of Rayleigh number on flow fields and temperature distributions for \( \phi = 60^\circ \), \( t = 0.033 \), and \( RK = 1 \). Figures 4(a) and 4(b) indicates that the flow consists of a one rotating vortex, which turned in clockwise. As Rayleigh number increased, the number of rotating vortex increased [Figures 4(c) and 4(d)]. For low Rayleigh number, conduction mode of heat transfer became dominant and isotherms were parallel to the top and bottom surfaces. The isotherms were distorted for higher Rayleigh number and second eddy was formed under the fin as seen in Figure 4(c). For the highest Rayleigh number, multiple circulation cells were formed in the enclosure. Maximum streamfunction values changed from 0.0131 to 60.3 for the Rayleigh number ranging from \( 10^3 \) to \( 10^6 \). The temperature gradient increased with the increase of Rayleigh number.

Figure 5 shows the effects of Rayleigh number on flow fields and temperature distributions for \( \phi = 150^\circ \), \( t = 0.033 \), and \( RK = 1 \). In this position of the fin, for low Rayleigh number, flow happened to turn in counterclockwise. Both maximum and minimum streamfunction values were almost the same. It means that for low Rayleigh number the position of the fin did not make any effect on heat and fluid flow. Three eddies were formed with increasing of Rayleigh number. In this position of the fin, absolute minimum values of streamfunction were lower for thicker fin. Effects of thermal conductivity ratio between fin and fluid are shown in Figure 6 at \( \phi = 60^\circ \), \( Ra = 10^5 \) and \( t = 0.1 \). The figure clearly shows that the thermal conductivity ratio mainly affected the minimum values of streamfunction. The isotherms were strongly affected by this ratio. Another effective parameter on fluid flow and temperature distribution was the thickness of the fin. This result is presented in Figure 7 for \( \phi = 60^\circ \), \( Ra = 10^5 \) and \( RK = 1 \). Based on the illustrated figure, the fin enlarged to the bottom side of the enclosure and flow under fin became motionless due to small volume. Thus, absolute values of minimum stream function became lower for thicker fin.
Figure 5  Flow patterns and constant temperature lines for 
$\phi = 120^\circ, t = 0.033, RK = 1$, (a) $Ra = 10^3$ (b) $Ra = 10^4$ 
(c) $Ra = 10^5$ (d) $Ra = 10^6$

$\Psi_{\text{max}}(0.529, 0.449) = 0.00925$  
$\Psi_{\text{min}}(0.9731, 0.9785) = -1.37e-6$

Figure 6  Flow patterns and constant temperature lines for 
$\phi = 60^\circ, Ra = 10^5, t = 0.1$, (a) $RK = 0.1$ (b) $RK = 10$

$\Psi_{\text{max}}(0.6059, 0.5941) = 18.6$  
$\Psi_{\text{min}}(0.6668, 0.1438) = -1.59$

Figure 7  Flow patterns and constant temperature lines for 
$\phi = 60^\circ, Ra = 10^5, RK = 1$, (a) $t = 0.1$ (b) $t = 0.2$

$\Psi_{\text{max}}(0.5974, 0.5899) = 19.9$  
$\Psi_{\text{min}}(0.4009, 0.1287) = -1.09$
Figure 8 Variation of mean Nusselt number with Rayleigh number, (a) for different thermal conductivity ratios at $t = 0.033, \phi = 1,200$ (b) for different thicknesses of the fin at $\phi = 1,200, RK = 1$ (see online version for colours)

Figure 9 Variation of mean Nusselt number with Rayleigh number for $t = 0.033, RK = 1$, (a) fin is inclined above vertical half of the cavity (b) fin is inclined below vertical half of the cavity (see online version for colours)

Figure 10 Variation of local Nusselt number with Rayleigh number for $t = 0.033, RK = 1, \phi = 60, Ra = 10^5$ (see online version for colours)

Figure 8(a) shows the mean Nusselt numbers calculated by using equation (10b) under the effects of thermal conductivity values. As seen from the figure, heat transfer increased with input energy (Rayleigh number) for all values of $RK$. The value was more effective for higher Rayleigh number due to domination of convective heat transfer. The mean Nusselt number became lower for $Ra = 10^4$ and higher for $Ra = 10^5$ and $5 \times 10^5$. Effects of thickness of the inclined fin on heat transfer are illustrated in Figure 8(b). The figure shows that heat transfer was almost the same for fin thickness of 0.1 and 0.033. Figure 9 compares the effects of inclination angle of the fin on mean Nusselt number. The figure clearly shows that the inclination angle of the fin was a control parameter for heat transfer. The highest heat transfer was formed for $\phi = 135^\circ$ and $150^\circ$. Figure 10 illustrates the variation of local Nusselt number for hot and cold walls. It is seen that heat transfer increased around the $X = 0.7$. However, heat transfer decreased around the $X = 0.35$ due to presence of the fin.

Vertical and horizontal velocity profiles are presented in Figures 11(a) and 11(b) at different location of the cavity ($\phi = 60^\circ, Ra = 10^5, RK = 1$ and $t = 0.033$). For $X = 0.75$ (far from the fin), sinusoidal velocity variation was formed. As seen from the Figure 11(a), the region under the fin behaved like single cavity. The maximum heat transfer occurred at $X = 0.50$. It means that the fin increased the flow velocity. For $Y = 0.50$ and $Y = 0.75$, the vertical velocity profiles did not affected from the presence of the fin as seen in Figure 11(b).
Figure 11 (a) Horizontal velocity profile (b) Vertical velocity profile for $r = 0.033, RK = 1, \phi = 60, Ra = 10^7$
(see online version for colours)

5 Conclusions

A numerical study was performed to examine the natural convection heat transfer and fluid flow in the inclined thick fin attached to a square enclosure. The governing equations in streamfunction-vorticity form were solved with finite difference technique and algebraic equations were solved using SUR method. Graphical results for the streamline and temperature contours for various parametric conditions were presented and discussed. The obtained results showed that inclination angle of the fin becomes a non-effective parameter for low Rayleigh numbers. However, both heat transfer and fluid flow were affected by the change of fin direction. The number of eddies changed according to the Rayleigh number. The fin was a perfect control element for local heat transfer on heated surface. Nusselt number was increased function of Rayleigh number. Heat transfer was increased with increasing thermal conductivity and Rayleigh number due to incoming energy in to the system. Multiple cells were formed at the studied Rayleigh number for each inclination angle. The inclination angle affects the flow strength and temperature distribution. Presence of inclined fin affects the heat transfer and fluid flow. Thus, it can be a control element. The inclined fin mostly controls the flow pattern than that of heat transfer. In other words, its hydrodynamic blockage mechanism is more effective. Furthermore, the flow field can be comparable with numerical work by using modern equipments and computations can be extended to turbulent flow.

References


Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$g$</td>
<td>Gravitational acceleration, m/s$^2$</td>
</tr>
<tr>
<td>$h$</td>
<td>Dimensionless fin location of $-y$ direction</td>
</tr>
<tr>
<td>$H$</td>
<td>Height of enclosure, m</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of enclosure, m</td>
</tr>
<tr>
<td>$Nu$</td>
<td>Mean Nusselt number</td>
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<td>$Pr$</td>
<td>Prandtl number</td>
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<tr>
<td>$Ra$</td>
<td>Rayleigh number</td>
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<tr>
<td>$RK$</td>
<td>Ratio of thermal conductivity, $k_s/k_f$</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature, K</td>
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<tr>
<td>$t$</td>
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<tr>
<td>$U$, $V$</td>
<td>Dimensionless axial and radial velocities</td>
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<tr>
<td>$X$, $Y$</td>
<td>Non-dimensional coordinates</td>
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Greek letters

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>$\alpha$</td>
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</tr>
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<td>$\beta$</td>
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<td>$\nu$</td>
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<tr>
<td>$\theta$</td>
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