Natural convection for hot materials confined within two entrapped porous trapezoidal cavities

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A B S T R A C T
This paper analyzes the detailed heat transfer and fluid flow within two entrapped porous trapezoidal cavities involving cold inclined walls and hot horizontal walls. Flow patterns and temperature distribution were obtained by solving numerically the governing equations, using Darcy’s law. Results are presented for different values of the governing parameters, such as Darcy-modified Rayleigh number, aspect ratio of two entrapped trapezoidal cavities and thermal conductivity ratio between the middle horizontal wall and fluid medium. Heat transfer rates are estimated in terms of local and mean Nusselt numbers. Local Nusselt numbers with spatial distribution exhibit monotonic trend irrespective of all Rayleigh numbers for the upper trapezoidal whereas wavy distribution of local Nusselt number occur for the lower trapezoidal.

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1. Introduction

Indirect heat transfer of fluids plays an important role in people’s lives [1–5]. It also has various applications in, for instance, material industries, geophysical processes, pollution control, food processing, etc. [6–9]. Heat exchangers have also been benefited in various environmental applications such as thermal pollution, air pollution, and water pollution. They are also very critical for energy conservation, conversion, recovery and successful implementation of new energy sources, and wide usage of it involves food processing, power transportation, air-conditioning and refrigeration, heat recovery, alternate fuels, etc. [10–16]. There exist two ways to analyze these applications: experimental and numerical methods. The latter is the most preferred way due to the high cost involved in experiments.

The present study analyzes the heat recovery of entrapped fluid between a stack of tubes, shown in Fig. 1a. The cold fluid passing through the stack of tubes may recover excessive heat associated with the hot fluid during the material processing. The application of the current study is given as follows: consider an assembly of diamond shaped tubes adjacent to each other and the space between two adjacent tubes is two porous trapezoidal cavities with entrapped fluid. Cold fluid may be pumped through the tubes to recover heat from hot entrapped fluid. This study examines the complete heat transfer in entrapped porous trapezoidal cavities in detail. Fig. 1b shows the computational domain with associated boundary conditions. The length of the bottom wall was \( L \) and height of the cavity was \( H = L/2 \).

Natural convection is important for thermal processing based on various applications [17]. In the literature, there are several studies on natural convection flows in porous trapezoidal cavities. The study for inclined trapezoidal enclosure at different inclination angles filled with a viscous fluid has been analyzed by Lee [18]. He made a numerical study to analyze the natural convection heat transfer in an inclined trapezoidal enclosure filled with a viscous fluid for different Prandtl numbers using body-fitted coordinate systems. It was shown that the heat transfer in a trapezoidal enclosure with two symmetrical, inclined sidewalls of moderate aspect ratio was a strong function of the orientation angle of the cavity. Kumar and Kumar [19] used parallel computation technique to analyze the natural convection heat transfer in a trapezoidal enclosure filled with a porous medium. The short bottom and the long top walls are taken adiabatic, while the sloping walls are differentially heated. They showed that the inclination of the side wall significantly affects the flow and temperature distribution. Baytas and Pop [20] solved to Darcy and energy equation in cylindrical coordinates using ADI method to analyze natural convection in a trapezoidal enclosure filled with a porous medium. It has been observed that up to Rayleigh number, \( Ra = 100 \), a conduction-dominated regime prevails, and afterwards a two-cellular convective flow regime takes place at the tilt angle 165°. Moukalled and Acharya [21] studied the conjugate natural convection in a trapezoidal enclosure with a divider attached inclined wall and filled with a viscous fluid. Moukalled and Darwish [22] made a numerical work on natural convection in a partitioned trapezoidal cavity using the special momentum-weighted interpolation method. They used conductive partition and showed that the presence of baffles decrease heat transfer as high as 70%. Other similar studies on natural convection in trapezoidal enclosures can be found in Peric [23], Van Der Eyden et al. [24], Boussaid et al. [25], Kumar [26], Papnicolaou and Belessiotis [27], Hammami et al. [28], Varol et al. [29–32], Natarajan et al. [33] and Basak et al. [34].
Basak et al. [35] recently made a numerical study solving the momentum and energy equations within two entrapped porous triangular cavities involving cold inclined walls and hot horizontal walls using a penalty finite element analysis with bi-quadratic elements. The authors also conducted other studies with two entrapped non-porous triangular cavities for different boundary conditions [36,37].

As seen above, there has been a considerable amount of work on heat transfer within trapezoidal and two entrapped triangular cavities reported in the literature. This study differs from the other studies because it incorporates two entrapped trapezoidal cavities. The objective of the present investigation is to analyze the heat recovery from hot fluids passing parallel to the hot plates and heat may be transported to the entrapped fluid between a stack of tubes. Cold fluid may be pumped through the tubes to recover heat from hot entrapped fluid. The present investigation aims to study the complete heat transfer details in two entrapped porous trapezoidal cavities (Fig. 1a). The computational domain with associated boundary conditions is shown in Fig. 1(a) and (b).

2. Physical model and governing equations

A schematic of the in two entrapped porous trapezoidal cavities and grid arrangement is shown in Fig. 1(a) and (b), respectively. The cold fluid is pumped through the hexagonal tubes. The flow rate may be sufficiently high such that cold fluid may act as a sink and the inclined wall is maintained at a constant cold temperature. The horizontal top and bottom walls are maintained hot. The constant temperature at the hot wall is due to the flow of hot gases over the top wall and below the bottom wall.

Governing equations are written as follows:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  
\[ \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = -\frac{g \beta K}{\nu} \frac{\partial T_f}{\partial x} \]  
\[ u \frac{\partial T_f}{\partial x} + v \frac{\partial T_f}{\partial y} = \alpha_m \left( \frac{\partial^2 T_f}{\partial x^2} + \frac{\partial^2 T_f}{\partial y^2} \right) \]

Greek letters
- \( \alpha_m \): effective thermal diffusivity of the porous medium
- \( \beta \): thermal expansion coefficient
- \( \theta \): non-dimensional temperature
- \( \nu \): kinematic viscosity
- \( \psi \): non-dimensional stream function

Subscript
- \( c \): cold
- \( f \): fluid
- \( h \): hot
- \( s \): solid

Fig. 1. a) Schematic diagram of the physical system and computational domain with the boundary conditions, and b) finite-difference grid for the computational domain.

Nomenclature
- \( AR \): aspect ratio of two entrapped trapezoidal cavities, \( lx/L \)
- \( g \): gravitational acceleration
- \( H \): height of the cavity, \( H = L/2 \)
- \( k_f \): thermal conductivity of the fluid
- \( k_s \): thermal conductivity of the middle horizontal wall
- \( k \): thermal conductivity ratio, \( k_s/k_f \)
- \( K \): permeability of the porous medium
- \( L \): dimensionless length of bottom or top horizontal walls
- \( lx \): dimensionless length of middle horizontal wall
- \( Nu_t \): local Nusselt number
- \( Nu \): mean Nusselt number
- \( Pr \): Prandtl number
- \( Ra \): Darcy-modified Rayleigh number
- \( T \): temperature
- \( u, v \): dimensionless axial and radial velocities
- \( X, Y \): non-dimensional coordinates

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and the energy equation for the middle horizontal wall are:

\[ \frac{\partial^2 T_s}{\partial x^2} + \frac{\partial^2 T_s}{\partial y^2} = 0. \] (4)

To write the above equations the assumptions are listed as follows:

• the properties of the fluid and the porous medium are constant,
• the cavity walls are impermeable,
• the Boussinesq approximation and the Darcy law model are valid,
• the viscous drag and inertia terms in the Darcy and Energy equations are negligible.

In equations, \( u \) and \( v \) are the velocity components along \( x \) and \( y \) axes, \( T_f \) is the fluid temperature, \( g \) is the acceleration due to gravity, \( T_s \) is the temperature of the solid partition wall, \( K \) is the permeability of the porous medium, \( \alpha_m \) is the effective thermal diffusivity of the porous medium, \( \beta \) is the thermal expansion coefficient and \( \nu \) is the kinematic viscosity. Introducing the stream function \( \psi \) defined as

\[ u = \frac{\partial \psi}{\partial y}; \quad v = -\frac{\partial \psi}{\partial x}. \] (5)

Eqs. (1)–(4) can be written in non-dimensional form as

\[ \frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = -Ra \frac{\partial \theta_f}{\partial X} \] (6)

\[ \frac{\partial \psi}{\partial X} \frac{\partial \theta_f}{\partial Y} - \frac{\partial \psi}{\partial Y} \frac{\partial \theta_f}{\partial X} = \frac{\partial^2 \theta_f}{\partial X^2} - \frac{\partial^2 \theta_f}{\partial Y^2} \] (7)

for the interface between lower and upper trapezoidal cavity

\[ \frac{\partial^2 \theta_s}{\partial X^2} + \frac{\partial^2 \theta_s}{\partial Y^2} = 0 \] (8)

for the middle horizontal wall, respectively. Here \( Ra = g\beta K(T_h - T_c)L/\alpha_m \nu \) is the Darcy-modified Rayleigh number for the porous medium and the non-dimensional quantities are defined as

\[ X = \frac{x}{L}; \quad Y = \frac{y}{L}; \quad \psi = \frac{\psi}{\alpha_m}; \quad \theta_f = \frac{T_f - T_c}{T_h - T_c}; \quad \theta_s = \frac{T_s - T_c}{T_h - T_c} \] (9)

The boundary conditions for streamline and temperature for the entrapped lower trapezoidal cavity are as follows (Fig. 1a),

\[ \psi = 0; \theta_f = 0 \text{ on } AB; \] (10a)

\[ \psi = 0; \theta_f = 0 \text{ on } EF; \]

\[ \psi = 0; \theta_f = 1 \text{ on } AF. \]

The boundary conditions for streamline and temperature for the entrapped upper trapezoidal cavity are as follows (Fig. 1a),

\[ \psi = 0; \theta_f = 0 \text{ on } BC; \] (10b)

\[ \psi = 0; \theta_f = 0 \text{ on } ED; \]

\[ \psi = 0; \theta_f = 1 \text{ on } CD. \]

Fig. 2. Comparison of streamlines and isotherms with literature: a) streamlines for present (left) and Basak et al. [35] (right), b) isotherms for present (left) and Basak et al. [35] (right).
Fig. 3. Streamlines (left) and isotherms (right) for different Darcy-modified Rayleigh numbers at $AR = 0.3$ and $k = 1$, a) $Ra = 100$, b) $Ra = 250$, c) $Ra = 500$, and d) $Ra = 1000$. 
for the interface between lower and upper trapezoidal cavity (Fig. 1a),

$$\Psi = 0; k_f \frac{\partial \theta_f}{\partial Y} = k_s \frac{\partial \theta_s}{\partial Y} \text{on BE};$$  \hfill (10c)

where $k_f$ and $k_s$ are the thermal conductivities of the fluid and middle horizontal wall, respectively. Physical quantities of interest in this problem are the local Nusselt number $N_u$, and the mean Nusselt number $\bar{Nu}$, which can be expressed as at the top horizontal wall:

$$N_u = - \left( \frac{\partial \theta_f}{\partial Y} \right)_{Y=1}$$

in the middle horizontal wall:

$$N_u = - \left( \frac{\partial \theta_f}{\partial Y} \right)_{Y=H}$$

at the bottom horizontal wall:

$$N_u = - \left( \frac{\partial \theta_f}{\partial Y} \right)_{Y=0} \quad , \quad Nu = \frac{1}{b} \int_b \bar{Nu} dX$$

Eqs. (6)–(8) were solved numerically with finite-difference method. Numerical simulations were carried out systematically in order to determine the effects of effective parameters of the problem as Darcy-modified Rayleigh Number $Ra$, thermal conductivity ratio between the middle horizontal wall and fluid medium $k(=k_s/k_f)$ and aspect ratio of two entrapped trapezoidal cavities, $AR = w/L$ on the flow and heat transfer characteristics. To solve the equations on inclined boundaries, the techniques of Asan and Namli [38] and Haese and Teubner [39] were followed. The used mesh treatment was depicted in Fig. 1(b). The inclined wall was approximated with staircase-like zigzag lines.

The iteration process was terminated when the following condition is satisfied

$$\sum_{ij} \left| \frac{\phi_{ij}^m - \phi_{ij}^{m-1}}{\phi_{ij}^{m-1}} \right| \leq 10^{-5}$$

where $m$ denoted the iteration step and $\phi$ stood for either $\theta_f, \theta_s$ or $\Psi$.

2.1. Validation of the code

For validation of the code, a study conducted by Basak et al. [35] was used. For the present study, for when $AR$ gets closer to zero ($AR \to 0(w \to 0)$), it turns out to be two entrapped triangular cavity, which allowed the researcher to compare the results with the ones in Basak et al. [35]. Results are shown by streamlines and isotherms contour plots in Fig. 2. The test shows that the results obtained using the present code give good agreement with those from the literature and it can be used with great confidence for further calculations.

Fig. 4. Streamlines (left) and isotherms (right) for different thermal conductivity ratios at $AR = 0.3$ and $Ra = 500$, a) $k = 0.1$ and b) $k = 10$.
Fig. 5. Streamlines (top) and isotherms (bottom) for different aspect ratios at $k = 1$, a) $AR = 0.2$ and b) $AR = 0.1$. 
3. Results and discussion

In this study, numerical results for streamlines, isotherms, local and mean Nusselt numbers for natural convection in two entrapped porous trapezoidal cavity were obtained for Darcy-modified Rayleigh number, aspect ratio of two entrapped trapezoidal cavities and thermal conductivity ratio between the middle horizontal wall and fluid medium.

Fig. 3(a) to (d) shows the streamlines (on the left) and isotherms (on the right) for different Darcy-modified Rayleigh number at $AR=0.3$ and $k=1$. In these figures, four eddies were formed from $Ra=100$ to 1000. Fluid near the center of the horizontal bottom wall moves towards the top of the lower trapezoidal whereas fluid near the inclined wall tends to go down for the upper trapezoidal, forming a pair of symmetric circulations in different directions for both the cavities. The fluid circulations follow clockwise direction in the right half of the axis of symmetry and counterclockwise direction in the left half of the axis of symmetry. As expected, values of stream function was higher for below eddies than that of above eddies for all Darcy-modified Rayleigh numbers. At low values of the Darcy-modified Rayleigh number, Fig. 3(a), the isotherms are found to be smooth, monotonic whereas for the lower trapezoidal, the isotherms are distorted near the cold walls and at the central regime. It may be noted that the maximum value of the stream function is 0.9 for the upper trapezoidal whereas that was 2.22 for the lower trapezoidal. This illustrates that the heat transfer is primarily due to conduction. Flow strength increases with increasing of the Darcy-modified Rayleigh number due to increasing of heat transfer from hot wall to cold wall. The maximum values of stream function for upper trapezoidal are changed as 1.9, 3.0 and 6.55 for $Ra=250$, 500 and 1000, respectively whereas for the lower trapezoidal, the maximum values of stream function are change as 1.75, 5.2 and 7.55. The stratification in isotherms has been observed in the upper trapezoidal for all the $Ra$ and $AR$. Thus, the influence of the $Ra$ and $AR$ for the fluid circulation within the upper trapezoidal is not significant. However, for $Ra=500$, the isotherms are compressed more at the middle horizontal wall for the lower trapezoidal and the temperature at the core varies within 0.5–0.6 for $AR=0.2$ whereas $\theta$ is found as 0.3–0.6 for $AR=0.1$.

Fig. 6(a) to (c) shows the variation of Nusselt number with the distance along the horizontal walls of the two entrapped trapezoidal cavities for different Darcy-modified Rayleigh numbers. As can be seen from the figures, distribution of local Nusselt number was completely symmetric according to middle axis of the all horizontal walls. In this context, Fig. 6(a) shows, due to high temperature gradients, the value of local Nusselt number is very high at the edges of the trapezoidal are changed as 0.7, 1.4 and 1.9 for $Ra=100$, 250 and 500, respectively whereas for the lower trapezoidal, the maximum values of stream function are change as 1.75, 5.2 and 7.55. The stratification in isotherms has been observed in the upper trapezoidal for all the $Ra$ and $AR$. Thus, the influence of the $Ra$ and $AR$ for the fluid circulation within the upper trapezoidal is not significant. However, for $Ra=500$, the isotherms are compressed more at the middle horizontal wall for the lower trapezoidal and the temperature at the core varies within 0.5–0.6 for $AR=0.2$ whereas $\theta$ is found as 0.3–0.6 for $AR=0.1$.

![Fig. 6. The variation of local Nusselt number along the horizontal walls for different Darcy-modified Rayleigh numbers at AR=0.3 and k=1. a) along the top wall, b) along the middle wall, and c) along the bottom wall.](image-url)
wall whilst the heat transfer rate decreases toward the center with nearly uniform values at the central region. The values of local Nusselt number increase monotonically with increasing of Darcy-modified Rayleigh number. The values of local Nusselt number are almost equal to each other for all Rayleigh numbers at $X = 0.1$ and $X = 0.9$. Fig. 6(b) shows the variation of Nusselt number with the distance along the middle horizontal wall. Values of local Nusselt number increase with increasing of Darcy-modified Rayleigh number and these values become constant along the middle horizontal wall. Fig. 6(c) shows the variation of Nusselt number with the distance along the bottom horizontal wall. It is observed that Nusselt number of the lower trapezoidal for $Ra = 100$, $250$ and $500$ parabolic variations. Wavy variation of local Nusselt number is obtained for $Ra = 1000$ due to increasing of convection heat transfer.

The overall heat transfer is presented via variation of mean Nusselt number and Darcy-modified Rayleigh number in Fig. 7 for different aspect ratios. As indicated in the figure, the mean Nusselt number is increased linearly with the increasing of the Darcy-modified Rayleigh number, which is an expected result. The value of mean Nusselt number becomes smaller with the increasing of aspect ratio because, in the case of higher aspect ratio, the length of the middle horizontal wall of the two entrapped trapezoidal cavity is increased. The highest mean Nusselt number value is obtained at the highest Darcy-modified Rayleigh number and the lowest value of aspect ratio.

4. Conclusions

Present study analyzes the details of natural convection heat transfer within two entrapped porous trapezoidal cavities which are enclosed between a pair of adjacent hexagonal tubes normally seen in heat recovery systems. The aim of this study is to analyze the efficient heat transfer details to the entrapped fluid in the system commonly used for practical applications in heat recovery during hot material processing. Equations of mass, momentum and energy have been written using Darcy law along with the Boussinesq approximation. Finite difference method was used to solve governing equations. The governing parameters were Darcy-modified Rayleigh number, aspect ratio of two entrapped trapezoidal cavities and thermal conductivity ratio between the middle horizontal wall and fluid medium. The conclusions derived from the present study may be listed as follows:

- Heat transfer increases with increasing of Darcy-modified Rayleigh number for all governing parameters.
- It is an interesting results that streamlines and isotherms contours become two entrapped porous trapezoidal cavities.
- The isotherms exhibit oscillatory pattern within the lower trapezoidal.
- The stratification in isotherms has been observed in the upper trapezoidal for all Darcy-modified Rayleigh numbers. Thus, the influence of the Darcy-modified Rayleigh number for the fluid circulation within the upper trapezoidal is not significant.
- The maximum mean Nusselt number is obtained for the highest Darcy-modified Rayleigh number and the lowest aspect ratio.

References


Fig. 7. Variation of mean Nusselt number with Darcy-modified Rayleigh number for different aspect ratios at $k = 1$. 


