Natural convection in divided trapezoidal cavities filled with fluid saturated porous media

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A B S T R A C T

A numerical work was performed to determine the heat transfer and fluid flow due to buoyancy forces in divided trapezoidal enclosures filled with fluid saturated porous media. In the present investigation, bottom wall was non-uniformly heated while two vertical walls were insulated and the top wall was maintained at constant cold temperature. The divider had constant thermal conductivity. Flow patterns and temperature distribution were obtained by solving numerically the governing equations, using Darcy’s law. Results are presented for different values of the governing parameters, such as Rayleigh number for a porous medium, location of the partition, thickness of the partition and thermal conductivity ratio between solid and fluid media. It was observed that the conduction mode of heat transfer became dominant inside the cavity for higher thickness of the partition, low Rayleigh numbers, and low thermal conductivity ratio.

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1. Introduction

Natural convection heat transfer and fluid flow in porous media was widely investigated in engineering applications such as post-accidental heat removal in nuclear reactors, solar collectors, drying processes, fire control, geophysics, and geothermal applications and so on [1–4]. The problem of buoyancy induced flow in trapezoidal cavity inclined at different angles filled with a viscous fluid was analyzed by Lee [5]. He performed a numerical study of this problem in order to analyze the behavior of the flow and heat transfer characteristics at different Rayleigh Ra and Prandtl numbers Pr using curvilinear geometry. It was shown that for Ra = 10^4 and Pr > 0.1, the heat transfer, in a trapezoidal enclosure with two symmetrical, inclined sidewalls of moderate aspect ratio, was a strong function of the orientation angle of the cavity. Non-isothermal boundary conditions could be seen in metallurgy to obtain regular melting for metal and it can be obtained with a cylindrical heater for laboratory experiment ([6], Saeid [7] and Varol et al. [8]).

Kumar and Kumar [9] analyzed the natural convection heat transfer in a trapezoidal cavity filled with a porous medium by using parallel computation technique. The bottom and top walls were taken as adiabatic and inclined walls were differentially heated. They showed that the inclination of the side wall was an important parameter on the flow and temperature distribution. Baytas and Pop [10] solved the Darcy and energy equations in cylindrical coordinates using ADI method in order to analyze natural convection in a porous trapezoidal enclosure. They indicated that up to Rayleigh number equal to 100, a conduction-dominated regime prevailed, and afterwards a two-cellular convective flow regime took place at the tilt angle 165°. Moukalled and Acharya [11] studied the conjugate natural convection heat transfer in a trapezoidal cavity with a divider attached onto inclined wall. In their case, the cavity was filled with a viscous fluid. Moukalled and Darwish [12] performed a numerical analysis of natural convection in a partitioned trapezoidal cavity using the special momentum-weighted interpolation method. They used conductive short partition/partitions and they showed that the presence of baffles decreased heat transfer as high as 70%. Other similar studies on natural convection in trapezoidal cavities were done by Peric [13], Van Der Eyden et al. [14], Boussaid et al. [15], Kumar [16], Papanicolaou and Belessiotis [17], Hammami et al. [18], Varol et al. [19–21] and Natarajan et al. [22].

The main aim of this study was to examine the natural convection heat transfer, temperature and flow field in a horizontally divided trapezoidal cavity filled with a fluid-saturated porous medium. Streamlines, isotherms, and local and mean Nusselt numbers will be presented in the following sections of the paper for different Rayleigh numbers, thermal conductivity ratio, thickness of the partition, and location of the partition.

2. Considered model

The physical model is drawn in Fig. 1 (a). This figure also shows the coordinates and boundary conditions. The system was a
3. Equations and their numerical solution

Governing equations are written as follows:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}
\]

\[
\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = -g\beta K \frac{\partial T_f}{\partial x} \tag{2}
\]

\[
\frac{u \partial T_f}{\partial x} + \frac{v \partial T_f}{\partial y} = \alpha_m \left( \frac{\partial^2 T_f}{\partial x^2} + \frac{\partial^2 T_f}{\partial y^2} \right) \tag{3}
\]

and the energy equation for the solid partition wall is:

\[
\frac{\partial^2 T_s}{\partial x^2} + \frac{\partial^2 T_s}{\partial y^2} = 0. \tag{4}
\]

To write the given equations the assumptions are listed as follows:

- the properties of the fluid and the porous medium are constant,
- the cavity walls are impermeable,
- the Boussinesq approximation and the Darcy law model are valid, and
- the viscous drag and inertia terms in the Darcy and energy equations are negligible.

In the equations, \( u \) and \( v \) are the velocity components along \( x \) and \( y \) axes, \( T_f \) is the fluid temperature, \( g \) is the acceleration due to gravity, \( T_s \) is the temperature of the solid partition wall, \( K \) is the permeability of the porous medium, \( \alpha_m \) is the effective thermal diffusivity of the porous medium, \( \beta \) is the thermal expansion coefficient and \( \nu \) is the kinematic viscosity. Introducing the stream function \( \psi \) defined as

\[
u = -\frac{\partial \psi}{\partial x}, \quad u = \frac{\partial \psi}{\partial y}.
\]
Eqs. (1–4) can be written in non-dimensional form as

\[ \frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -Ra \frac{\partial \theta_f}{\partial X} \]

for the fluid-saturated porous medium and

\[ \frac{\partial^2 \theta_f}{\partial X^2} + \frac{\partial^2 \theta_f}{\partial Y^2} = 0 \]

for the solid partition wall, respectively. Here \( Ra = g \beta L (T_h - T_c) / \alpha_m v \) is the Rayleigh number for the porous medium and the non-dimensional quantities are defined as

\[ X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad \Psi = \frac{\psi}{\alpha_m}, \quad \theta_f = \frac{T_f - T_c}{T_h - T_c}, \quad \theta_s = \frac{T_s - T_c}{T_h - T_c}. \]

The boundary conditions of Eqs. (6–8) are:

for all solid boundaries

\[ \Psi = 0; \]

on the bottom wall (hot), non-uniform temperature is applied as

\[ \theta_f = \lambda [1 - \cos(2\pi X)] \]

where the value of \( \lambda \) was taken as 0.5;

on the top wall (cold),

\[ \theta_f = 0; \]

on the side walls (adiabatic),

\[ \frac{\partial \theta_f}{\partial n} = 0; \]

for the interface between solid partition wall and porous medium,

\[ k_f \frac{\partial \theta_f}{\partial Y} = k_s \frac{\partial \theta_s}{\partial Y} \]

where \( k_f \) and \( k_s \) are the thermal conductivities of the fluid and solid walls, respectively. Physical quantities of interest in this problem are the local Nusselt number \( Nu \) and the mean Nusselt number \( Nu_m \), which can be expressed as

<table>
<thead>
<tr>
<th>Inclination angle of the side wall (( \gamma ))</th>
<th>15°</th>
<th>30°</th>
<th>45°</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Nu_m ) (Baytas and Pop [25])</td>
<td>2.95</td>
<td>2.62</td>
<td>2.23</td>
</tr>
<tr>
<td>( Nu_m ) (present study)</td>
<td>2.872</td>
<td>2.585</td>
<td>2.217</td>
</tr>
</tbody>
</table>
on the bottom wall:

\[ Nu = \left( -\frac{\partial \theta}{\partial Y} \right)_{Y=0}, \quad Nu_m = \frac{1}{\ell} \int_0^\ell Nu \, dX; \quad (11a, b) \]

under surface of the partition:

\[ Nu = \left( -\frac{\partial \theta}{\partial Y} \right)_{Y=h-t/2}, \quad Nu_m = \frac{1}{\ell X} \int_0^{\ell X} Nu \, dx; \quad (12a, b) \]

upper surface of the partition:

\[ Nu = \left( -\frac{\partial \theta}{\partial Y} \right)_{Y=h+t/2}, \quad Nu_m = \frac{1}{\ell X u} \int_0^{\ell X u} Nu \, dx \quad (13a, b) \]

for the interface between the solid partition and the porous medium.

Eqs. (6–8) were solved numerically with finite-difference method. Numerical simulations were carried out systematically in order to determine the effects of effective parameters of the problem as Rayleigh number \( Ra \), thermal conductivity ratio \( k = k_s/k_f \), dimensionless thickness of the solid partition wall \( t' = t/H \) and dimensionless location of partition in the y-direction \( h' = h/H \) on the flow and heat transfer characteristics. To solve the equations on inclined boundaries, the technique of Asan and Namli [23] and Haese and Teubner [24] were followed. The used mesh treatment was depicted in Fig. 1(b). The uppermost grid-point on each vertical grid line coincided with the top wall of the trapezoidal enclosures. The inclined wall was approximated with staircase-like zigzag lines. A series of tests on grid-independency against the streamlines and isotherms were conducted for \( t = 0.2, h = 0.45, k = 1 \) and \( Ra = 1000 \). They are presented in Fig. 2 by streamlines (on the left) and isotherms (on the right). As seen from this figure, the grid sizes of \( 101 \times 36, 201 \times 71 \) and \( 301 \times 106 \) were selected. Iteration numbers also increase with increasing of mesh size. The comparison of figures indicated that there were no big differences on streamlines, isotherms and stream functions among grid dimensions. Thus, the

Fig. 3. Streamlines (left) and isotherms (right) for different Rayleigh numbers at \( t = 0.1 \) and \( h = 0.45 \): a) \( Ra = 50 \), b) \( Ra = 250 \), c) \( Ra = 500 \), d) \( Ra = 1000 \).
grid dimension was considered as $101 \times 36$. The iteration process was terminated when the following condition was met

$$
\sum_{i,j} \frac{\Phi_{m}^{i,j} - \Phi_{m-1}^{i,j}}{\Phi_{m}^{i,j}} \leq 10^{-5}
$$

(14)

where $m$ denoted the iteration step and $\Phi$ stood for either $\theta, \theta_s$ or $\Psi$.

### 3.1. Validation of the code

To validate the study, the obtained results were validated against the existing results for a square cavity filled with a porous medium as a first comparison. Thus, the comparison of the present results for the mean Nusselt number $Nu_{tm}$ as defined by Eq. (11b), with those from the open literature was made for a value of $Ra = 1000$. For this, the comparison results can be found in our earlier publications as Varol et al. [8]. The second test was performed to show validation of the code with literature as given in Table 1. In this case, results are compared with the results reported by Baytas and Pop [25], which were for a parallelogram cavity filled with a fluid-saturated porous medium at different inclination angles. As seen from Table 1, the maximum difference between the results by Baytas and Pop [25] and the present results is 0.5%. Thus, the two tests show that the results of the present code are in good agreement with those from the open literature and they can be used with great confidence for further calculations.

### 4. Results and discussion

In this study, numerical results for streamlines, isotherms, local and mean Nusselt numbers for natural convection in partially divided porous trapezoidal cavity were obtained for $Ra$ numbers, thickness of the horizontal partition, location of the horizontal partition, and thermal conductivity ratio.

Fig. 3 (a) to (d) shows the streamlines (left) and isotherms (right) for different Rayleigh numbers at $t = 0.1$, $h = 0.45$ and $k = 1$. In these figures, four eddies were formed from $Ra = 50$ to 1000. These eddies were distributed symmetrically according to middle vertical axis of the cavity. Also, absolute values of streamfunctions were the same at the left and right eddies. It is noticed that eddies above and below of
Fig. 5. Streamlines (left) and isotherms (right) for different dimensionless locations of the partition at $t = 0.1$, $k = 1$ and $Ra = 500$: a) $h = 0.15$, b) $h = 0.30$, c) $h = 0.60$, d) $h = 0.75$.

Fig. 6. Streamlines (left) and isotherms (right) for different thermal conductivity ratios at $t = 0.2$, $h = 0.45$, and $Ra = 500$: a) $k = 0.1$, b) $k = 1$, c) $k = 10$. 
the partition turned different directions. The right eddy above the fin turned towards clockwise direction. However, the right eddy below the partition turned in counterclockwise. As expected, absolute values of streamfunction were higher for below eddies than that of above eddies for lower Rayleigh number. In this figure, the right column shows the temperature distribution. With increasing of Rayleigh number, the plumelike flow was formed from bottom to top. On the contrary, bowl shaped distribution was observed.

Fig. 4 (a) to (d) illustrates the streamlines (left) and isotherms (right) to show the effects of thickness of partition on flow fields and temperature distribution. The values of thickness changed from \( t = 0.05 \) to \( t = 0.5 \). Increasing of thickness of partition enhanced the domination of conduction mode of heat transfer which was an expected result. Flow strength became weaker with increasing of thickness of partition. As seen from the figure, the number of eddies was independent from the thickness. Convection mode of heat transfer became stronger with increasing of thickness of partition. The reason of this phenomenon was transfer of heat from bottom to top. This can be seen from the isotherms clearly.

The effect of dimensionless location of the partition on flow fields and temperature distribution is shown in Fig. 5 (a) to (d). The location of partition changed from 0.15 to 0.75. The figure shows that the shape of eddies strongly depended on the location of the partition. The flow strength decreased above the partition and increased below.

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**Fig. 7.** The variation of local Nusselt numbers along the horizontal walls (along the bottom wall (left), along the bottom side of the partition (middle), and along the upper side of the partition (right)): a) for different Rayleigh numbers at \( t = 0.1, \ h = 0.45, \) and \( k = 1 \); b) for different thermal conductivity ratios at \( t = 0.2, \ h = 0.45, \) and \( Ra = 500 \); c) for different dimensionless thicknesses of the partition at \( h = 0.45, \ k = 1, \) and \( Ra = 500 \); and d) for different dimensionless locations of the partition at \( t = 0.1, \ k = 1, \) and \( Ra = 500 \).
the partition with increasing of location of the partition. It was an interesting result that the direction of eddy was the same at left and right for minimum value of location. For this case, plumelike distribution of isotherms was formed for the whole cavity. On the contrary, bowl shaped distribution was formed for higher value of location of the partition.

Fig. 6 (a) to (c) illustrates the effects of thermal conductivity for \( t = 0.2 \), \( h = 0.45 \) and \( Ra = 500 \) on flow fields and temperature distribution by plotting streamlines and isotherms, respectively. The results were given for the values of thermal conductivity from \( k = 0.1 \) to \( k = 10 \). In the case, the flow strength increased with increasing of thermal conductivity values above the partition. More flow was heated under the partition. For \( k = 0.1 \), the partition behaved as an insulation material. It was an interesting result that flow strength became weaker for higher values of thermal conductivity.

Fig. 7 (a) presents the variation of the local Nusselt number along the bottom wall (left), along the bottom side of the partition (middle) and along the top side of the partition (right) for different Rayleigh numbers at \( t = 0.1 \), \( h = 0.45 \), and \( k = 1 \). In the same manner, the local Nusselt number was presented for different values of thermal conductivity. As can be seen from the figures, distribution of local Nusselt number was completely symmetric according to the middle axis of the bottom wall. Due to domination of conduction mode of heat transfer, bell-shaped distribution was formed at \( Ra = 50 \). However, there was a minimal point at the middle of the bottom wall due to the motionless fluid at that part. On the bottom wall of the trapezoidal enclosure, heat transfer increased with increasing of thermal conductivity as seen from Fig. 7 (b) (on the left). Heat transfer was almost constant for \( k = 0.1 \) along the bottom and top sides of the partition. Fig. 7 (c) and (d) presents the local Nusselt number for different thicknesses of the partition and location of the partition, respectively. As seen from these figures, the local Nusselt number increased with decreasing of the thickness of the partition due to domination of the convection mode of heat transfer. There was no strong effect of location of the partition on heat transfer at the bottom and top sides of the partition as given in Fig. 7 (d).

Mean Nusselt numbers are presented in Fig. 8 (a) to (f). Actually, the left and right columns can be compared in the same segment in order to clarify the results. Thus, Fig. 8 (a) and (d) gives effects of thermal conductivity on heat transfer with Rayleigh number. The figures indicate that heat transfer increased with increasing of Rayleigh number depending on thermal conductivity. Heat transfer has an increasing function of thermal conductivity. Thus, more heat was transferred from bottom to top. It was noticed that heat transfer became constant for higher values of thermal conductivity. Fig. 8 (b) and (e) can be compared with each other to show the effects of...
partition thickness on heat transfer. In this case, \( k \) was taken as a fixed value \( (k = 1) \). For higher thickness of the partition, the heat transfer regime was almost at conduction. For low values of Rayleigh number, thickness of the partition became insignificant. Fig. 8 (c) and (f) shows the effects of location of partition for different Rayleigh numbers. The figure displays that the location of partition parameter became insignificant for low values of Rayleigh number. For \( h = 0.30 \), heat transfer had a minimal value around \( Ra = 250 \). However, when the partition was located near the top or bottom wall, higher heat transfer occurred around the same value of Rayleigh number. As an interesting result, the heat transfer had a maximum point for \( h = 0.45 \) for higher Rayleigh numbers. It means that heat transfer was mostly with convection when the partition was located at the middle of the cavity. Globally, the results showed that increasing of Rayleigh number enhanced the heat transfer for the same effective parameters.

5. Conclusions

In this numerical work, natural convection in a partially divided porous trapezoidal cavity for a wide range of the governing parameters for non-uniformly heated wall conditions was examined. In the view of the results presented, the main findings can be summarized as follows:

- Heat transfer increases with increasing of Rayleigh number and thermal conductivity ratio.
- Location of the partition becomes insignificant for low values of the Rayleigh number.
- Location of partition is more effective when the partition is located near either the top or bottom wall.
- The heat transfer decreases with increasing of partition thickness due to domination of conduction mode of heat transfer.
- For huge values of thermal conductivity, heat transfer becomes constant inside the enclosure.
- The partition is a control parameter for flow field and temperature distribution. Number of eddies is an independent parameter from all effective parameters. However, their turns or changes in their rotation depend on the parameters.

References