

Flow Simulation of GO Nanofluid Inside Semi Porous Channel

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Abstract

In this study, the analytical solution of the graphene oxide-water nanofluid inside semi permeable channel has been presented. The Reconstruction of Variational Iteration Method (RVIM) has been used to compute an approximate solution of the nonlinear differential equation. Comparisons are made between the numerical solution and the results of the RVIM. The effects of different parameters, especially the effect of the solid volume fraction of graphene oxide nanoparticle on velocity are investigated and shown graphically. The dynamic viscosity and the effective thermal conductivity of the nanofluid have been approximated by the Brinkman and Maxwell-Garnetts models respectively.

Key words: Graphene Oxide, Nanofluid, Flow Simulation, RVIM.

1. Introduction

The flow of Newtonian and non-Newtonian fluids in a porous surface channel has become very important because of its applications in biophysical flows, e.g., pulsating diaphragms, filtration, blood flow, artificial dialysis, binary gas diffusion, and air and blood circulation in the respiratory system.

As a new type heat transfer fluid, nanofluid (i.e., the mixture of host fluid and nanoparticles) can be used to enhance the heat transfer of heat exchangers. The nanoparticle size has significant effects on the thermophysical properties of nanofluid [1–3] and the convective heat transfer characteristics of nanofluid [4–6], so it may have effects on the overall performance of the heat exchangers using nanofluid.

Graphene was found to display high quality electron transport at room temperature. Theoretical study was performed on determination of thermal conductivity of graphene and suggests that it has unusual thermal conductivity [7].

After that an experimental study was carried out on determination of thermal conductivity of graphene and 5300 W/mK was measured for thermal conductivity of single layer graphene. [8] Nanofluids are having wide area of application in electronic and cooling industry. Hydrogen exfoliated graphene (HEG) dispersed deionized (DI) water, and ethylene glycol (EG)

based nanofluids were developed by Tessa Theres et, al [9]. Thermal conductivity and heat transfer properties of these nanofluids were systematically investigated. A 0.05% volume fraction of f-HEG dispersed DI water based nanofluid shows an enhancement in thermal conductivity of about 16% at 25°C and 75% at 50°C. The enhancement in Nusselts number for these nanofluids is more than that of thermal conductivity.

Most phenomena in real world are described through nonlinear equations. Some of the equations are solved by using numerical methods, whereas the others by employing the analytic methods such as Differential Transformation Method (DTM) [10], Homotopy Perturbation Method (HPM) [11] and Reconstruction of Variational Iteration Method (RVIM)[12].

The purpose of this paper is to study the flow of GO-water nanofluid inside semi porous channel.

2. Material and Methods

In this section the mathematical formulation of the problem and solution procedure will be presented.

2.1. Problem formulation

Consider the laminar 2D stationary flow of a viscous fluid in semi porous channel made by a long rectangular plate. It is assumed that one of the walls is permeable and the other is without transpiration.

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0, \quad (1)$$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho_{nf}} \frac{\partial P^*}{\partial x^*} + \quad (2)$$

$$\frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} \right) - u^* \frac{\sigma_{nf} B^2}{\rho_{nf}} + \quad (3)$$

$$u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = -\frac{1}{\rho_{nf}} \frac{\partial P^*}{\partial y^*} + \quad (3)$$

$$\frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}} \right)$$

Auxiliary conditions can be specified such as:

$$\begin{aligned} y^* = 0 : u^* &= u_0^*, v^* = 0 \\ y^* = h : u^* &= 0, v^* = -q \end{aligned} \quad (4, 5)$$

Calculating a mean velocity U by the relation:

$$U \times h = \int_0^h u^* \times dy^* = L_x \times q \quad (6)$$

We consider the following transformations:

$$x = \frac{x^*}{L_x}; y = \frac{y^*}{h}, \quad (7)$$

$$u = \frac{u^*}{U}; v = \frac{v^*}{q}, P_y = \frac{P^*}{\rho_f \cdot q^2} \quad (8)$$

Then, we can consider two dimensionless numbers: the Hartman number for the description of the magnetic forces [13] and the Reynolds number for the dynamic forces

$$Ha = Bh \sqrt{\frac{\sigma_f}{\rho_f \cdot \nu_f}}, \quad (9)$$

$$Re = \frac{hq}{\mu_{nf}} \rho_{nf}. \quad (10)$$

Quantity ε is defined as the ratio between distance h and a characteristic length L_x of the slider. This ratio is normally small. Berman's similarity transformation is used to be free from the aspect ratio ε :

$$v = -V(y), u = \frac{u^*}{U} = u_0, U(y) + x \frac{dV}{dy} \quad (11)$$

Introducing Eqs. (6) and (10) into Eqs. (1) and (3) leads to the dimensionless equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (12)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\varepsilon^2 \frac{\partial P_y}{\partial x} + \quad (13)$$

$$\frac{\mu_{nf}}{\rho_{nf}} \frac{1}{hq} \left(\varepsilon^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - u \frac{Ha^2 B^*}{Re A^*}$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial P_y}{\partial x} + \quad (14)$$

$$\frac{\mu_{nf}}{\rho_{nf}} \frac{1}{hq} \left(\varepsilon^2 \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$$

Using Maxwell model, the effective density (ρ_{nf}) can be defined as [12]:

$$\rho_{nf} = \rho_f (1 - \phi) + \rho_s \phi \quad (15)$$

In above equation, ϕ is the solid volume fraction of nanoparticles. The dynamic viscosity of the nanofluids given:

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}} \quad (16)$$

The effective thermal conductivity of the nanofluid can be approximated by the Maxwell-Garnett (MG) model as [12]:

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \quad (17)$$

The effective electrical conductivity of nanofluid was presented by Maxwell [12] as below:

$$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3\left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi}{\left(\frac{\sigma_s}{\sigma_f} + 2\right) - \left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi} \quad (18)$$

Based on above equations, A^* and B^* are constant parameters:

$$A^* = (1 - \phi) + \frac{\rho_s}{\rho_f} \phi$$

$$B^* = 1 + \frac{3\left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi}{\left(\frac{\sigma_s}{\sigma_f} + 2\right) - \left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi} \quad (19)$$

Introducing Eq. (11) in the second momentum equation (13) shows that quantity $\partial P_y / \partial y$ does not depend on the longitudinal variable x . With the first momentum equation, we also observe that $\partial^2 P_y / \partial x^2$ is independent of x . We omit asterisks for simplicity. Then a separation of variables leads to [13]:

$$V'^2 - VV'' - \frac{1}{\text{Re}} \frac{1}{A^*(1 - \phi)^{2.5}} V'' + \frac{Ha^2}{\text{Re}} \frac{B^*}{A^*} V' = \varepsilon^2 \frac{\partial P_y}{\partial x^2} \quad (20)$$

$$= \varepsilon^2 \frac{1}{x} \frac{\partial P_y}{\partial x^2}$$

$$UV' - VU' = \frac{1}{\text{Re}} \frac{1}{A^*(1 - \phi)^{2.5}} \quad (21)$$

$$[U' - Ha^2 B^* (1 - \phi)^{2.5} U]$$

The right-hand side of Eq. (15) is constant. So, we derive this equation with respect to y . This gives

$$V^{IV} = Ha^2 V'' + \text{Re}[VV'' - VV'''] \quad (22)$$

where primes denote differentiation with respect to y and asterisks have been omitted for simplicity. The dynamic boundary conditions become

$$y = 0 : U = 1, V = 0, V' = 0$$

$$y = 1 : U = 0, V = 1, V' = 0 \quad (23, 24)$$

2.2. Solution procedure

In the following section, an alternative method for finding the optimal value of the Lagrange multiplier by the use of the Laplace transform will be investigated. suppose x, t are two independent variables, consider t as the principal variable and x as the secondary variable. if $u(x, t)$ is function of two variables x and t , when the Laplace transform is applied with t as a variable, definition of Laplace transform is:

$$L(u(x, t); s) = \int_0^\infty e^{-st} u(x, t) dt \quad (25)$$

We have some preliminary notations as:

$$L\left(\frac{\partial u}{\partial t}; s\right) \quad (26)$$

$$= \int_0^\infty e^{-st} \frac{\partial u}{\partial t} dt = sU(x, s) - u(x, 0)$$

$$L\left(\frac{\partial^2 u}{\partial t^2}; s\right) \quad (27)$$

$$= s^2 U(x, s) - sU(x, s) - u_t(x, 0)$$

$$U(x, s) = L(u(x, t); s) \quad (28)$$

We often come across functions which are not the transform of some known function but then they can possibly be as a product of two

function. Thus we may be able to write the given function as $U(x, s)$, $V(x, s)$ where $U(s)$ and $V(s)$ are known to the transform of the function $u(x, t)$, $v(x, t)$, when the Laplace transform is applied to t as a variable, respectively; then $U(x, s)$, $V(x, s)$ is the Laplace Transform of

$$\int_0^t u(x, t - \varepsilon) v(x, \varepsilon) d\varepsilon : \quad (29)$$

$$L^{-1}(U(x, s), V(x, s))$$

$$= \int_0^t u(x, t - \varepsilon) v(x, \varepsilon) d\varepsilon$$

To facilitate our discussion of Reconstruction of Variational Iteration Method (RVIM), introducing the new linear or nonlinear function $h(u(x, t)) = f(x, t) - N(u(x, t))$ and considering the new equation, rewrite $h(u(x, t)) = f(x, t) - N(u(x, t))$ as:

$$L(u(t, x)) = h(t, x, u) \quad (30)$$

Now, for implementation the correctional function of VIM based on new idea of Laplace transform, applying Laplace Transform to both sides of the above equation so that we introduce artificial initial conditions to zero for main problem, then left hand side of equation after transformation is featured as:

$$L[L\{u(x, t)\}] = U(x, s)P(s) \quad (31)$$

Where $P(s)$ is polynomial with the degree of the highest order derivative of linear operator:

$$L[L\{u(x, t)\}] \quad (32)$$

$$= U(x, s)P(s) = L[h\{(x, t, u)\}]$$

$$U(x, s) = \frac{L[h\{(x, t, u)\}]}{P(s)} \quad (33)$$

Suppose that $D(s) = 1/P(s)$, Using the convolution theorem, Taking the inverse Laplace transform on both side of Equation.(32),

$$u(x, t) = \int_0^t d(t - \varepsilon) h(x, \varepsilon, u) d\varepsilon \quad (34)$$

$$u_0(x, t) + \int_0^t d(t - \varepsilon) h(x, \varepsilon, u) d\varepsilon \quad (35)$$

And $u_0(x, t)$ is initial solution with or without unknown parameters. In absence of unknown parameters, $u_0(x, t)$ should satisfy initial boundary conditions.

3. Results and Discussions

In this section, we will discuss about the obtained results of Graphene Oxide-water nanofluid flow inside semi-porous channel for various Reynolds and Hartman number. The physical properties of GO- water nanofluid are tabulated in Table.1.

Table1. Thermo physical properties of water and GO nanoparticle [12]

Material	$\rho(kg / m^3)$	$C_p(j / kgk)$	$k(W / m.k)$
Pure Water	997.1	4179	0.613
Graphene Oxide	1800	717	5000

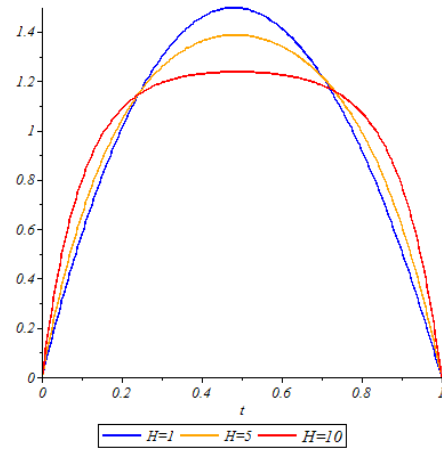


Figure 1. $V'(y)$ for various Hartman number when $Re = 1$, $\phi = 0.1$

Figure 1. shows the effect of Hartman number on $V'(y)$ in case Reynolds number is fixed. It is illustrated that the increment of Ha may decrease the $V'(y)$ in constant Reynolds number and solid volume fraction.

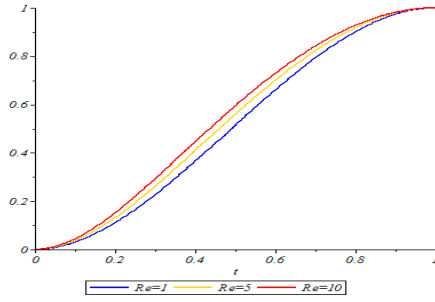


Figure 2. $V(y)$ for various Hartman number when $Ha = 1$, $\phi = 0.1$

$V(y)$ for various Reynolds number have been presented in Figure 2. Results shows by increasing Reynolds number, $V(y)$ will be increased.

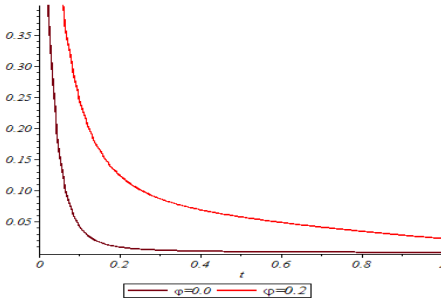


Figure 3. $U(y)$ for various solid volume fraction when $Ha = 1$, $Re = 10$

Figure 3 illustrates the effect of solid volume fraction of graphene oxide nanoparticles on $U(y)$ when Hartman number and Reynolds number are constant. As it can be seen the increment of solid volume fraction causes increment in $U(y)$.

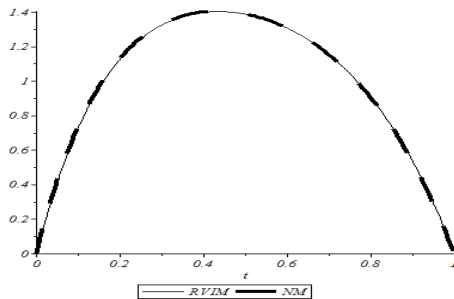


Figure 4. Comparison of analytical results and numerical ones when $Ha = 5$, $Re = 5$, $\phi = 0.05$ ($V'(y)$)

In Figure 4 a proper comparison is made between the numerical solution obtained by Runge Kutta method and RVIM. A great agreement between analytical solutions and numerical ones are illustrated.

4. Conclusions

This paper concerns the analytical investigation of the GO-water nanofluid flow in a semi-porous channel. The Similarity Berman's transformation is employed to convert the governing partial differential equations of a steady laminar flow of an electrically conducting fluid in a two dimensional channel. Reconstruction of Variational Iteration Method (RVIM) has been used to obtain the expressions for velocity fields.

Graphs are sketched and discussed for various parameters, especially the effect of the expansion ratio on velocity fields. the results indicated that the Reynolds number, Hartmann number and solid volume fraction have strong effect on velocity boundary layer thickness.

5. References

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