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Design of Optimal and Combinational Logic Circuit Using Karnaugh Method in Quantum Circuits

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Abstract
Today, quantum computers are still in development and are an area actively studied by researchers. Although quantum computers have high potential, there are significant technical difficulties and obstacles to achieving practical applications. Many research labs and companies are working on increasing the number of qubits, solving stability problems, and improving error correction techniques, and scalability. These efforts aim at advances such as the development of more stable and reliable qubits, the discovery of more precise control and reading techniques, and the design of more advanced algorithms. This study aims to establish a connection between quantum computers and classical computers. To achieve this, research and work have employed a Karnaugh-based approach. Karnaugh maps serve as highly useful tools in the analysis of complex logic circuits. Depending on one's needs and design requirements, different logic functions can be analyzed and simplified using Karnaugh maps. In this study, the Karnaugh map-based method is utilized to obtain the Boolean function from the state table of the Quantum circuit generated from the RCViewer+ synthesis tool. Subsequently, the obtained function is transformed into a combinational logic circuit. As a result, an optimal combinational circuit corresponding to the quantum circuit can be obtained.

Keywords: Quantum Circuits, Karnaugh Method , Quantum, Combinational Logic Circuit.

1. Introduction

Quantum computers are potentially revolutionary devices that offer a different computing paradigm from traditional computers [1]. These computers, based on quantum mechanics, have the potential to perform some calculations faster and more effectively than classical computers cannot. Quantum computers are made up of basic units called quantum bits or "qubits" [2]. Qubits have the ability to be the same in both 0 and 1 states, thanks to a phenomenon called superposition, unlike ordinary bits that can only take a value of 0 or 1. The state of superposition grants quantum computers the capability to concurrently process vast amounts of information in parallel. The most remarkable aspect of quantum computers is their potential to achieve a substantial speedup in solving certain computational problems by leveraging phenomena such as quantum parallelism and quantum entanglement [3]. In particular, quantum computers can have a significant impact on solving complex mathematical problems such as factorization. This can have significant consequences for deciphering and breaking cryptographic systems. But, quantum computer development and deployment still face significant technical challenges. Stability, fault tolerance, data readability, and scalability of qubits are still areas of study and improvement. In addition, the development of algorithms, software, and hardware tools for the widespread use of quantum computers is an important step. In short, quantum computers are devices that offer different computing power and potential than traditional computers, but still need development and have significant technical challenges. Their potential has the potential to make a huge impact in areas such as cryptography, optimizing, and molecular modeling. However, further research and development is required for quantum computers to become widespread.

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Literature Summary

In recent years, existing studies and articles in the literature related to quantum technology, in general, have also been seen in research on topics such as cost calculation, fault tolerance[4], and quantum optimization. In addition, there are many studies on reversible logic [5]. Reversible logic is a vital topic today and has applications as diverse as low-power CMOS, quantum computing, nanotechnology, cryptography, optical computing, DNA computing, digital signal processing (DSP), quantum dot cellular automata, communications, and computer graphics. There are many studies in the literature related to these application areas. Gaur et al. [6] compared the operating costs of quantum gates in their study. Thus, they contributed to future studies on the operating cost of common doors. In addition, 3*3 MCT and MCF gates are also explained and used in this study [6]. Roy et al. [7] presented an implementation of a reversible ALU (Arithmetic Logic Unit). This application demonstrates a reversible ALU that uses the MRI (Modified Reversible Gate) gate and the HNG (Hybrid Nanometric Gate) gate to generate six logical computations [7]. In Rather et al. [8], a low quantum cost 4×4 reversible gate TKG (Assembly of Reversible Gates) was presented, and then a 4-bit reversible multiplier was designed. In the proposed study, a new 4×4 reversible gate, TKG, is proposed. TKG was used in both designs. Her work has made a significant contribution to trade [8]. Anwar et al. [9], proposed a new 4*4 reversible RR (Reed-Muller) gate. The reduced latency and quantum cost of a reversible universal shift register using a gate was calculated. The proposed design was functionally tested and verified [9]. Garipelli et al. [10], introduced basic reversible logic gates that can perform more complex operations using reversible circuits designed for more complex systems [10]. Mia et al. [11], some reversible logic gates that can carry out more complex operations using data and quantum computers in the design of more complex systems with reversible circuits have been studied. The article shows data on reversible gates that help design complex circuits [11]. Thakral et al. [12], discussed the proposed new reversible logic gates and their quantum implementation using the RCViewer+ tool. The methods used were seen as Toffoli Networking Methodology, Quick Guide for RCViewer/RCViewer+ Tool, and Quantum Implementation of Reversible Logic Gates [12]. The continuation of the literature summary is presented in Table 1 in tabular form.

Table 1. Historical development and results of studies in the literature on the synthesis of quantum reversible circuits

Scientific Study	Year	Dataset	Method	Result	Synthesis Tool
Nagamani et al.[13]	2015	Vedic math techniques Toffoli, Peres and Fredkin gates	4x4 Urdhva Tiryakbhayam Multiplier design	QC: 143 CL(Ancilla): 32 GO:28 UD:34δ TRLIC: 240	RCViewer+ Decompose Circuit Tool
Misra et al.[14]	2017	QCA field in nanoelectronics	QCADesigner DC gate	QC:%25 GC:%66 GO%50	RCviewer +
Sultana et al.[15]	2017	16 units 4 variable gates	Toffoli mapping Reversible Comparison	Ort QC:24,5 Ort GC:9,25 Ort TG:23,25	RCViewer+
Misra et al.[16]	2018	CNOT, NOT, CV	R-CQCA Flipflops BCD	QC: %36,66 R-CQCA:%65.49	RCVeiwer+
Thakral et al.[17]	2018	Fredkin gate	Quantum Application of Fredkin Gate	QC: 7 GC: 7 TG: 7	RCViewer+

**T-depth: Depth of gate is a measure of how many 'layers' of t-gate are executed in parallel, t-number: total number of t-gates in the quantum circuit, Qubit cost: Qubit Cost is Total Qubits Required to Design the Quantum. DFT: Design for Several Testability Using the Method of Converting a Standard Circuit to Its Testable Form, QCA: Quantum Dot Cellular Automaton, GC: Gate Count, QC: Quantum Cost, UD: Unit Delay, LC: Logical Computing, GO: Garbage Output, CI: Fixed Input, QCA: Binary Decoder, Low-Cost Module, BEDesigner: 1D Codec with Binary Low Cost, BEDesigner: 1DQD, BG-2: Binary Gray, GB-2: Gray Binary, NG-R1: New Gate = New Gate, R = Reversible, TG: Two-Qubit Gate Number, R-CQCA: Reversible Conservative Quantum Cellular Automaton

Motivation

Generally, combinational logic circuits are used for conventional computers and electronic devices. Quantum circuits, on the other hand, play an important role as a potential technology of the future in the fields of quantum

information processing and computation. Classical circuits focus on providing deterministic and error-free results. Quantum circuits, on the other hand, enable parallel operations and potentially faster and more efficient computations using quantum properties such as superposition and aliasing. There are still significant technical challenges to the development, and deployment of quantum circuits. At the same time, there isn't much study and research on the connection between classical circuits and quantum circuits. This situation provided motivation for the proposed study.

Combinational Logic Circuits

Combinational logic circuits are electronic circuits that combine electronic components to perform a specific logical operation. Computers, cell phones, televisions, digital clocks, and many other electronic devices contain logic circuits. Logic circuits perform logical operations based on two basic logical values (true and false, often denoted as "1" and "0"). These operations can be expressed using layers of boolean algebra principles and basic expenditure operations (such as and, or, not). Combinational logic circuits are built with a combination of basic structures known as connectors. Logical elements produce results on behaviors, uses, and outputs applied to inputs "Example logical elements encompass gates (such as AND, OR, XOR), flip-flops, multiplexers, and decoders.[18] The design and analysis of combinational logic circuits constitute a vital subject in electronic engineering and computer science. Advanced combinational logic circuits are tailored to fulfill diverse requirements like high performance, low power consumption, and optimized functionality. These circuits serve as the fundamental building blocks for processors, memory units, buses, and other intricate digital systems."

2. Material

In the study, the RCViewer+ simulator and Karnaugh map were used. The Karnaugh map is preferred because it offers significant advantages in the design of smaller and simpler logic circuits. In addition, the study of the combinatorial logic circuit equivalent of the quantum circuit also provides the connection between them by using the truth table.

Rcviewer+ Synthesis Tool

RCViewer is a simple application software written in C++.TFC format [12]. An explanation of the array indexes given in Table 2 indicates what the codes and indexes in the ".tfc" file given in Figure 1 mean. It is also used in the design of reversible circuits and parameter extraction [19].

```
.v A,B,C,D
.i A,B,C,D
.o A,B,C,D
BEGIN
t3 A,C,D
t3 C,B,A
t3 A,B,D
t2 B,A
t2 C,B
t1 B
t2 B,D
END
```

Figure 1. Example TFC File

The '.tfc' format in Figure 1 is explained below.
"A, B, C, D": Shows the list of all variables in the network.
".v A, B, C, D": Shows the list of input variables in the network.
".i A, B, C, D": Shows the list of output variables in the network.
".o A, B, C, D": Input constant in the network
BEGIN: Indicates the network start of the gateway.
In the lines after BEGIN, the doors are arranged according to the prescribed format.

END: Indicates the network end of the gateway.
In electronics and computer science, "gates" are basic elements used for information processing and performing logic operations in digital logic circuits. These gates produce outputs by applying certain logical operations relative to the inputs. Conventional digital logic gates are used in classical circuits while quantum gates are used in quantum circuits. The ".tfc" format of the gates in quantum circuits is shown in Table 2.

Table 2. Display of doors in ".tfc" format [12]

GATE		TFC Format
NOT gate	TOF(Φ;a)	t1 a
Cnot gate	TOF(a;b)	t2 a,b
CCNOT gate	TOF(a,b;c)	t3 a,b,c
SWAP gate	FRE(Φ;a,c)	F2 a,b
FREDKİN gate	FRE(a;b,c)	F3 a,b,c
V Gate		V
V^+ Gate		V^+

Karnaugh Map-Based Method

Karnaugh Map is a tool used to simplify boolean algebra and design logic circuits [20]. It is named after the American mathematician Maurice Karnaugh. The K-map consists of cells arranged in a grid and representing combinations of boolean variables. The purpose of the Karnaugh Map is to express a boolean function in a simpler form. It optimizes the design of the combinatorial logic circuit. These maps visually represent the "1" and "0" values of the boolean functions. It makes it easy to find a simplified expression by grouping similar combinations. Using K-maps, boolean equations can be found in logic tables or circuit diagrams. Karnaugh maps are generally used for the analysis of functions with 2, 3 or 4 inputs. It uses a quadrilateral map for a two-input function, an octagonal map for a three-input function, and a hexadecimal map for a four-input function. Each cell is associated with neighboring cells on the map. A simpler expression is obtained by combining similar neighboring cells.
The basic usage steps of Karnaugh Map are:

I. The truth table of the boolean function is created.
II. The size of the Karnaugh Map is determined (2x2, 4x4, etc.), this size depends on the number of variables.
IV. By grouping the "1" cells in the map, their cells are combined to obtain a simplified expression.
V. The simplified expression is expressed in boolean algebra notation that represents the original boolean function.

Examples of Some Commonly Used Basic Logic Gates in the Karnaugh Map

The examples in Figure 2 show how some logic functions with different input numbers are represented by Karnaugh maps. These are just a few examples, and larger Karnaugh maps can be used for functions with more inputs.

NOT Function (1 input):

A	Exit
0	1
1	0

NOR Function (2 inputs):

A\B	0	1
0	1	0
1	0	0

AND Function (2 inputs):

A\B	0	1
0	0	0
1	0	1

3-input AND Function:

A\B\C	00	01	11	10
0	0	0	0	0
1	0	1	1	1

OR Function (2 inputs):

A\B	0	1
0	0	1
1	1	1

3-input OR Function:

A\B\C	00	01	11	10
0	0	1	1	1
1	1	1	1	1

XOR Function (2 inputs):

A\B	0	1
0	0	1
1	1	0

3-input XOR Function:

A\B\C	00	01	11	10
0	0	1	1	0
1	1	0	0	1

NAND Function (2 inputs):

A\B	0	1
0	1	1
1	1	0

4-input NAND Function:

A\B\C	00	01	11	10
0	0	1	1	0
1	1	0	0	1

Figure 2. Karnaugh map representation of logic functions

3. Proposed Method

In the study, the proposed method consists of three steps, these are quantum, Karnaugh-Map, and classical circuit steps. The block diagram of the proposed method is shown in Figure 3. Later in this section, these steps are explained in detail.

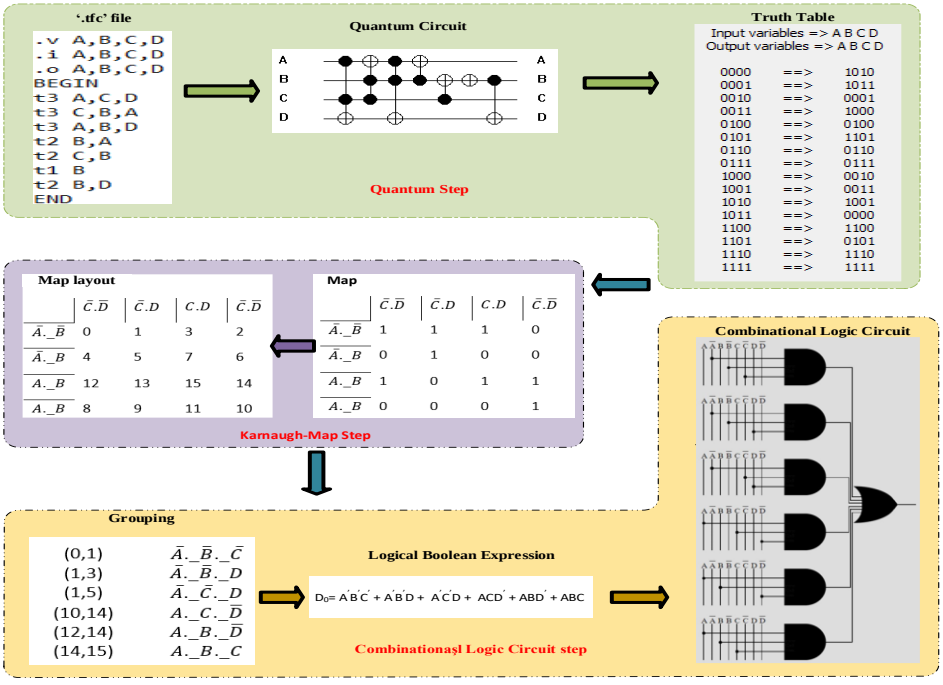


Figure 3. Block diagram of the proposed method

Quantum Step

Quantum circuit design involves arranging and controlling logic gates (quantum gates), which are used to make quantum computers work and solve certain computational problems. Quantum circuits differ from conventional computer circuits in that quantum gates represent gates with properties unique to quantum mechanics, rather than classical logic gates, and are designed using a variety of synthesis tools. In the quantum step of the study, a sample ".tfc" file is created, as can be seen in Figure 4, and the ".tfc" file is run with the RCViewer+ Tool installed on classical computers and converted into a quantum circuit. The truth table of the Quantum circuit consisting of four inputs, four outputs, and seven gates is
taken.

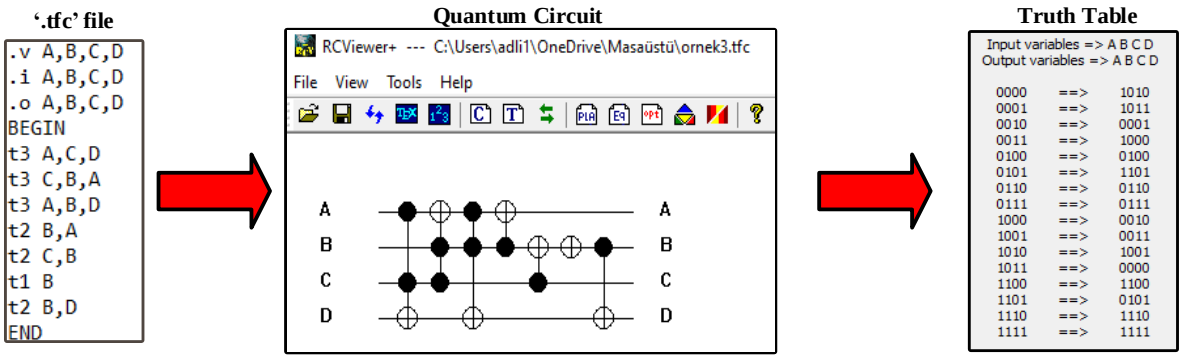
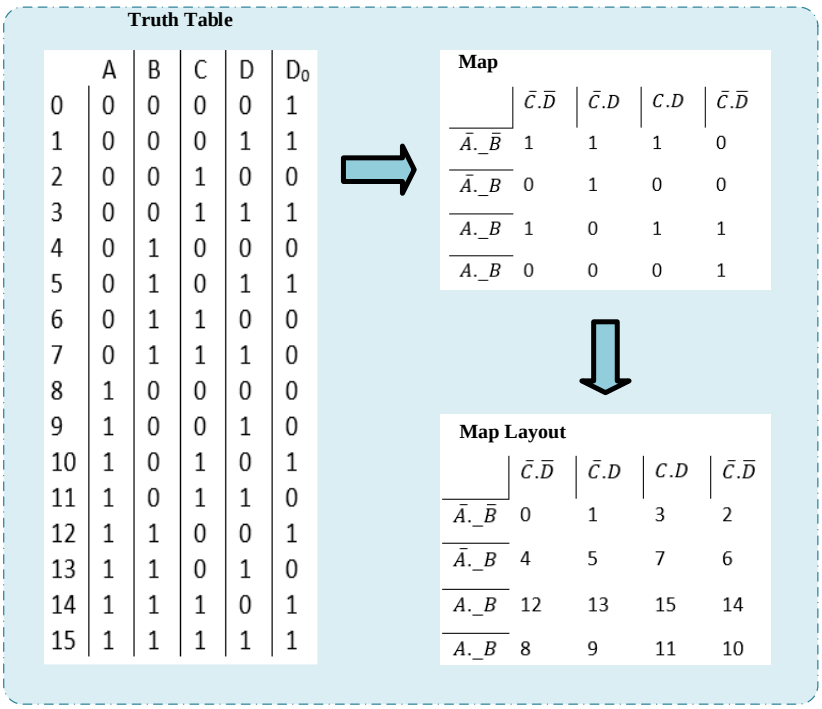


Figure 4. The result of the quantum circuit design and truth table of the sample ".tfc" file

Karnaugh-Map Step

In the Karnaugh step, as seen in Figure 5, the Karnaugh-Map-based method is applied to the truth table obtained in the quantum step. In practice, one of the 4-input A, B, C, and D variables, D output, whose value is equal to 1, is taken and written in place of D0. A hexadecimal map is then used for a four-input function. Each cell is associated with neighboring cells in the map, and a simpler expression is obtained by combining similar neighboring cells. The map layout is created by replacing the values corresponding to the bits.



	A	B	C	D	D_0
0	0	0	0	0	1
1	0	0	0	1	1
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	0
14	1	1	1	0	1
15	1	1	1	1	1

	$\bar{C}.\bar{D}$	$\bar{C}.D$	$C.D$	$\bar{C}.\bar{D}$
$\bar{A}._\bar{B}$	1	1	1	0
$\bar{A}._B$	0	1	0	0
$A._B$	1	0	1	1
$A._\bar{B}$	0	0	0	1

	$\bar{C}.\bar{D}$	$\bar{C}.D$	$C.D$	$\bar{C}.\bar{D}$
$\bar{A}._\bar{B}$	0	1	3	2
$\bar{A}._B$	4	5	7	6
$A._B$	12	13	15	14
$A._\bar{B}$	8	9	11	10

Figure 5. Using karnaugh-Map from the sample state table

Combinational Logic Circuit Step

A hexadecimal map is then used for a four-input function. Each cell is associated with neighboring cells in the map, and a simpler expression is obtained by combining similar neighboring cells. The map layout is created by replacing the values corresponding to the bits.

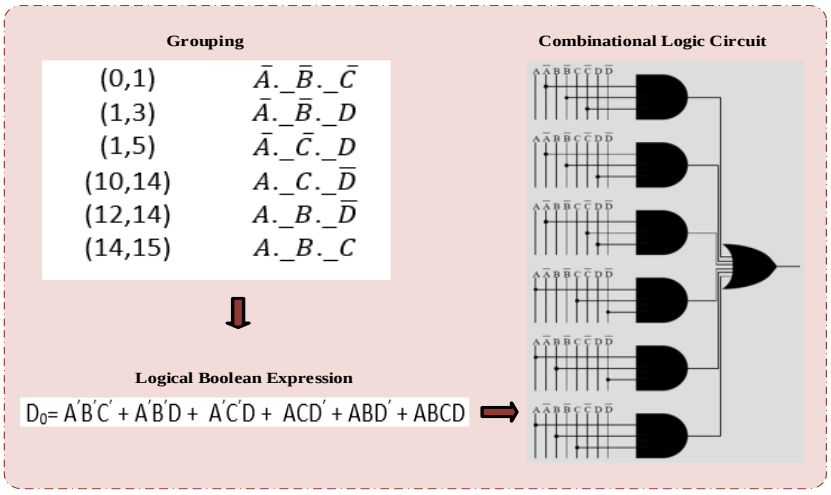


Figure 6. Classical circuit generation from the resulting logical expression

4. Conclusion

Because quantum circuits are built on a different paradigm than combinatorial logic circuits, they should not be applied directly, like the traditional Karnaugh map. The design and optimization of quantum circuits are carried out with special algorithms and techniques that take into account the properties of quantum mechanics. However, as can be understood from the explanations and studies, the Karnaugh map provides advantages such as simplifying the operation structures, reducing the delay times and facilitating the debugging process. The connection of quantum circuits with combinational logic circuits has been tested. It has also been found that it can be used to quickly analyze complex logic functions with a large number of inputs and outputs. It is foreseen that the use and development of this method in the future will contribute to the study. The Karnaugh map is a viable and effective method only for a certain number of entries (2, 3, or 4 entries). Other techniques and methods may need to be used for logical expressions with more inputs.

5. Acknowledgment

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