

Effects of piezo-viscous dependency on squeeze film between circular plates: Couple Stress fluid model

U. P. SINGH

Ansal Technical Campus, Faculty of Applied Science, Department of Mathematics, Lucknow; India;
*journals4phd@gmail.com

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Abstract

In high pressure fluid flows applications such as fluid film lubrication, microfluidics and geophysics, the piezo-viscous effect i.e. viscosity-pressure dependence plays an important role. In the present theoretical investigation, the combined effects of piezo-viscous dependency and non-Newtonian couple stresses on the performance of circular plate squeeze film bearings have been investigated using Stokes Micro-continuum theory of couple stress fluids together with the exponential variation of viscosity with pressure. Analytic solution for film pressure is obtained using small perturbation analysis. The numerical results for pressure and load capacity with different values of viscosity-pressure parameter are calculated and compared with iso-viscous couple stress and Newtonian lubricants. Due to piezo-viscous effect, enhanced pressure, increased load capacity and longer response time is observed in the analysis.

Key words: Couple stress Fluids, Load-carrying capacity, Piezo-viscous effect, Squeeze-film lubrication, Viscosity-pressure

1. Introduction

The squeeze film lubrication plays very important role in various applications of engineering and technology. Ball bearings, matching gears, machine tools, rolling elements and automotive engines are some common examples of squeeze film lubrication. The squeeze film phenomenon is also observed during approach of faces of disc clutches under lubricated condition. The mechanical action (squeezing, shearing etc.) changes the lubricants' temperature as well as viscosity and density which account for the variation of bearings performance characteristics. Many researchers such as Dowson [1], Wada and Hayashi [2] and Kapur [3] emphasized the variation of viscosity and density with temperature and pressure and reported a significant variation in the pressure, load capacity etc. In high pressure applications, the viscosity of lubricants depends much on pressure than the temperature [4]. In such mechanism, the variation of viscosity is also more significantly over the density variation [4, 5]. Denn [6] emphasized that under a pressure of about 5 MPa, the dependence of viscosity on pressure become important even if the flow is incompressible. Bair et al. [7] shown that the viscosity of bis(phenoxyphenoxy)benzene varies

from 0.0251 Pa.s to 72 Pa.s under pressure variation 0.1–300 MPa, and the viscosity of dipentaerythritol hexaisostearate varies from 0.0251 Pa.s to 66.2 Pa.s under pressure variation 0.1-942 MPa under isothermal condition. Therefore, it is reasonable to analyze the bearing performance with a pressure dependent viscosity [8] considering the isothermal, incompressible flow of lubricants of the form

$$\mu = \mu_o e^{\beta p} \quad (1)$$

where μ is the viscosity, P is the pressure, μ_o is the viscosity at atmospheric pressure, and β is the pressure-viscosity coefficient.

Kottke [9] reported the range of β between 10 to 70 MPa⁻¹ for lubricants. Venner and Lubrecht [10] reported that β may range between 10⁻⁸ to 2×10⁻⁸Pa⁻¹ for mineral oils.

The investigation of squeeze film characteristics between rectangular, porous rectangular, curved circular plates etc. has been analyzed from time to time by various researchers such as Burbidge and Colin [11] Sanni [12], Abell and Ames [13], Gupta and Vora [14], Gupta and Gupta [15], Murti [16] and Wu [17] using classical Newtonian model. After Stokes [18] proposed micro-continuum theory for couple stresses in fluids accounting the particle size effects of micro- structures in

lubricants, many researchers investigated the effect of couple stresses on the squeeze film characteristics of various systems. Manivasakan and Sumathi [19] studied the effect of couple stress on squeezing films between curved circular plates. Nadumani et al. [20, 21] studied the couple stress effect on squeezing action between stepped circular plates and sphere-plate system with rough surface. Lin et al [22,23] analyzed the squeeze film characteristics between two different spheres, wide parallel-plate squeeze film, and the cylinder-plate squeeze film. Alkough and Der-Fa [24] studied the non-Newtonian effects on Rayleigh step-bearings. Ishizawa [25] analyzed the unsteady flow between two parallel disks with varying gap.

As discussed earlier, the viscosity-pressure dependence is important while studying the high pressure phenomena in lubrication. In recent years, researchers also focused their attention to account the viscosity-pressure dependence to analyze the lubrication phenomenon. Gould [26] analyzed the squeezing characteristic in high pressure spherical system and reported an enhanced characteristics due variation of viscosity with pressure. Yadav and Kapur [3] reported an enhanced pressure and load capacity of hydrostatic thrust bearings considering the viscosity variation with pressure and temperature. Kalogirou [7] analyzed incompressible Poiseuille flows of Newtonian liquids with a pressure-dependent viscosity. Lu and Lin [27,28] investigated the viscosity-pressure dependence on sphere plate squeezing system and wide parallel rectangular pelate with non-Newtonian couple stresses lubricant and reported an enhanced film pressure and load capacity due to piezo-viscous effect.

However, the piezo-viscous effect on the squeezing characteristics between parallel circular plates with non-Newtonian couple stress effect was not considered by any of the authors, which is considered in the present theoretical analysis.

2. Analysis

A systematic diagram of squeeze film lubrication between circular plates approaching

each other with a normal velocity $\frac{dh}{dt}$ is shown in Fig. 1. The lubricant in the system is considered as non-Newtonian incompressible couple stress fluid [18]. The body forces and body couples are assumed to be absent in the analysis.

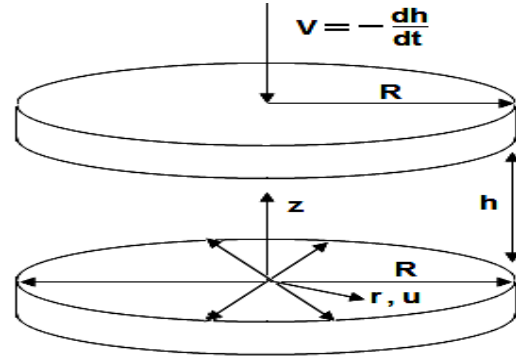


Figure 1. Systematic diagram of circular plate squeeze film bearing.

Under the assumptions of thin film lubrications [1], the constitutive equations in cylinder polar coordinates governing the motion of steady laminar flow of a couple stress fluid in the film region are -

$$\frac{1}{r} \frac{\partial}{\partial r} (\bar{r} \bar{u}) + \frac{\partial \bar{w}}{\partial \bar{z}} = 0 \quad (2)$$

$$\mu \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} - \eta \frac{\partial^4 \bar{u}}{\partial \bar{z}^4} = \frac{\partial \bar{p}}{\partial \bar{r}} \quad (3)$$

$$\frac{\partial \bar{p}}{\partial \bar{z}} = 0 \quad (4)$$

which are solved under the following boundary conditions :

$$\bar{u} = \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} = 0 \quad \text{and} \quad \bar{w} = 0 \quad \text{at} \quad \bar{z} = 0 \quad (5)$$

$$\bar{u} = \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} = 0 \quad \text{and} \quad \bar{w} = -\frac{\partial \bar{h}}{\partial \bar{r}} \quad \text{at} \quad \bar{z} = \bar{h} \quad (6)$$

where $\bar{u}$ and $\bar{w}$ are the velocity components in $\bar{x}$ and $\bar{z}$ directions, $\bar{h}$ is the film thickness between the bearings plates, μ is viscosity given by equation (1.1) and η is new material constant responsible for couple stresses in the fluids.

Taking the dimensionless quantities as

$$p = \frac{\bar{p} \bar{h}_o^3}{\mu_o R^2 \left(\frac{d\bar{h}}{d\bar{t}} \right)}, \quad r = \frac{\bar{r}}{R}, \quad z = \frac{\bar{z}}{h_o},$$

$$u = \frac{\bar{u} h_o}{R \left(\frac{d\bar{h}}{d\bar{t}} \right)}, \quad w = \frac{\bar{w}}{\left(\frac{d\bar{h}}{d\bar{t}} \right)},$$

$$\alpha = \frac{\beta \mu_o R^2 \left(\frac{d\bar{h}}{d\bar{t}} \right)}{\bar{h}_o^3}, \quad h = \frac{\bar{h}}{h_o}, \quad \mu = \frac{\bar{\mu}}{\mu_o}$$

and $v = \frac{1}{h_o} \sqrt{\frac{\eta}{\mu_o}}$

the equations (1-4) takes the dimensionless form

$$\mu = e^{\alpha p} \quad (7)$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r u) + \frac{\partial w}{\partial z} = 0 \quad (8)$$

$$\mu \frac{\partial^2 u}{\partial z^2} - v^2 \frac{\partial^4 u}{\partial z^4} = \frac{\partial p}{\partial r} \quad (9)$$

$$\frac{\partial p}{\partial z} = 0 \quad (10)$$

and the related boundary conditions (5-6) as

$$u = \frac{\partial^2 u}{\partial z^2} = 0 \text{ and } w = 0 \text{ at } z = 0 \quad (11)$$

$$u = \frac{\partial^2 u}{\partial z^2} = 0 \text{ and } w = -1 \text{ at } z = h \quad (12)$$

Solving equations (9) under the relevant boundary conditions (11-12), the radial velocity profile is obtained as

$$u = \frac{1}{2\mu} \frac{\partial p}{\partial r} \left(z^2 - z h \right) + \frac{v^2}{\mu^2} \frac{\partial p}{\partial r} \left[1 - \frac{\cosh \left(\frac{\sqrt{\mu}}{v} \left(z - \frac{h}{2} \right) \right)}{\cosh \left(\frac{\sqrt{\mu}}{v} \frac{h}{2} \right)} \right] \quad (13)$$

Integrating equation (8) for w with the relevant boundary conditions (11-12) using equations (7) and (13), the modified Reynolds equation is obtained as

$$\frac{d}{dr} \left(r f(h, \alpha, v, p) \frac{dp}{dr} \right) = -12r \quad (14)$$

where

$$f(h, \alpha, v, p) = h^3 e^{-\alpha p} - 12h v^2 e^{-2\alpha p} + 24v^3 e^{-\frac{5}{2}\alpha p} \tanh \left(\frac{h}{2v} e^{\frac{1}{2}\alpha p} \right) \quad (15)$$

Due to the non-linear nature of the modified Reynolds equation (14), the classical perturbation technique is adopted to find the

close form solution for the pressure distribution.

Since the piezo-viscous parameter α is small ($\alpha \ll 1$) [10], the pressure p can be written as

$$p = p_o + \alpha p_1 + \alpha^2 p_2 + \dots \quad (16)$$

Substituting equation (16) in the modified Reynolds equation (14) and leaving the terms with the second and higher powers of α , the following simplified equations are obtained as-

$$\frac{d}{dr} \left(r \frac{dp_o}{dr} \right) = -\frac{12r}{f_o} \quad (17)$$

$$\frac{d}{dr} \left(r \frac{dp_1}{dr} \right) = -\frac{f_1}{f_o} \frac{d}{dr} \left(r p_o \frac{dp_o}{dr} \right) \quad (18)$$

Solving the equations (17) and (18) under the boundary conditions

$$\frac{dp}{dr} = 0 \text{ at } r = 0 \quad (19)$$

$$p = 0 \text{ at } r = 1 \quad (20)$$

the following perturbed solution are obtained

$$p_o = \frac{3}{f_o} (1 - r^2) \quad (21)$$

$$p_1 = -\frac{9}{2} \frac{f_1}{f_o^3} (1 - r^2)^2 \quad (22)$$

where

$$f_o = h^3 - 12h v^2 + 24v^3 \tanh \left(\frac{h}{2v} \right) \quad (23)$$

$$f_1 = -h^3 + 30h v^2 - 60v^3 \tanh \left(\frac{h}{2v} \right) - 6h v^2 \tanh^2 \left(\frac{h}{2v} \right) \quad (24)$$

The load carrying capacity of the bearing can be calculated as follows

$$\bar{W} = 2\pi \int_0^R \bar{r} \bar{p} d\bar{r} \quad (25)$$

which takes the dimensionless form

$$W = \frac{h_o^3}{2\pi \mu R^4 \frac{d\bar{h}}{d\bar{t}}} \bar{W} = \int_0^1 r p dr \quad (26)$$

The response time-height relation can be calculated by integrating the equation (26) under the condition $\bar{h} = h_o$ at $\bar{t} = 0$ (i.e., $h=1$ at $t=0$) as follows

$$t = -\frac{\bar{w} h_o^2}{2\pi \mu R^4} \bar{t} = \int_h^1 \left(\int_0^1 r p dr \right) dh \quad (27)$$

Since, the integrand of t is a nonlinear function of h , Numerical integration has been performed to calculate time-height relation using *Mathematica 7.0*.

3. Results and Discussion

In the present paper, the combined influence of couple stresses and the piezo-viscous dependency on the squeeze film bearing characteristics have been predicted on the basis of Stokes couple stress fluid theory [18] together with the exponential variation of viscosity with pressure [8] avoiding the inertia and cavitation effects.

The piezo-viscous effect is analyzed with a viscosity-pressure parameter α on one hand and the effects of couple stresses are analyzed using a dimensionless parameter $\nu = \frac{1}{R} \sqrt{\frac{\eta}{\mu_0}}$ on the other, where $\sqrt{\frac{\eta}{\mu_0}}$ has the dimension of length and it can be identified as the function of molecular length of polar additives in a non-polar lubricant. In figure 2 to figure 7, the results of pressure, load capacity and response time are compared for piezo-viscous and iso-viscous lubricants.

The ranges of parameters are taken as : piezo-viscous parameter $\alpha = 0.02$ and couple stress parameter $\nu = 0, 0.05, 0.1, 0.15$.

Figure 2 shows the variation of dimensionless pressure p with respect to the dimensionless radius r for film thickness $h = 0.05$. On comparison with the iso-viscous Newtonian case ($\nu = 0$), the pressure with iso-viscous couple stress is higher and it increases with increase of couple stress parameter ν for each value of the radius. It establishes the validity of present analysis for iso-viscous couple stress lubricant. Further, it is observed that for each value of couple stress parameter, the pressure with piezo-viscous analysis is significantly higher than the pressure with iso-viscous analysis. However, the relative variation of pressure with piezo-viscous effect increases with the increase of couple stress parameter. It proves the significance of viscosity-pressure variation for couple stress lubricants.

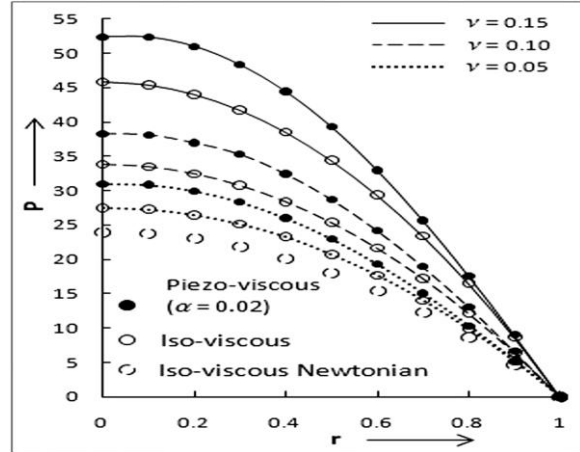


Figure 2. Variation of dimensionless pressure with respect to dimensionless radius for iso-viscous and piezo-viscous lubricants.

Figure 3 shows the variation of dimensionless maximum film pressure with respect to the dimensionless film thickness h . It is observed that the maximum film pressure with couple stress lubricants is higher than that in Newtonian case. The maximum pressure also increases with the increase of couple stress parameter ν . Again, for each value of couple stress parameter, the maximum pressure in piezo-viscous case is higher than that in the iso-viscous case.

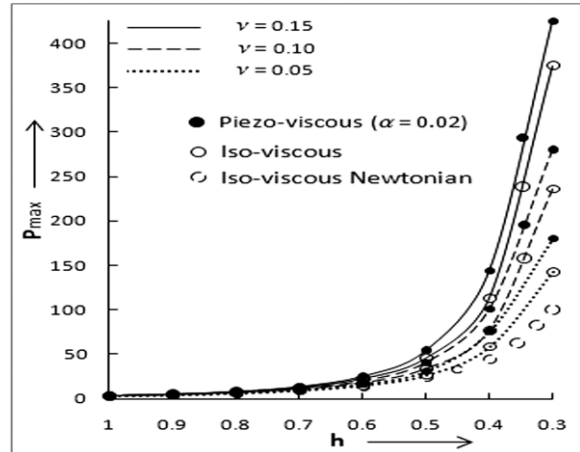


Figure 3. Variation of dimensionless maximum pressure with respect to film thickness for iso-viscous and piezo-viscous lubricants.

Figure 4 shows the variation of dimensionless load capacity with respect to the dimensionless film thickness h . It is observed that the load capacity obtained with the iso-

viscous couple stress lubricants is higher than that of iso-viscous Newtonian lubricants. The load capacity also increase with the increase of couple stress parameter. Further, for each value of couple stress parameter, the piezo-viscous effect increases the load capacity significantly. A sequential change from iso-viscous Newtonian to iso-viscous couple stress and iso-viscous to piezo-viscous couple stress increases the load capacity systematically.

Figure 5 shows the variation of dimensionless load capacity with respect to the piezo-viscous parameter (α) for dimensionless film thickness $h=0.5$. In both the cases of Newtonian and Couple stress lubricants, the load capacity increases with the increase of piezo-viscous parameter α . However, the relative change in the load capacity is observed to increase with the increase of couple stress parameter. The values of the load capacity for iso-viscous case are observed to be lowest. Again, the load capacity for couple stress is observed to be higher than the Newtonian and increases with the increase of couple stress.

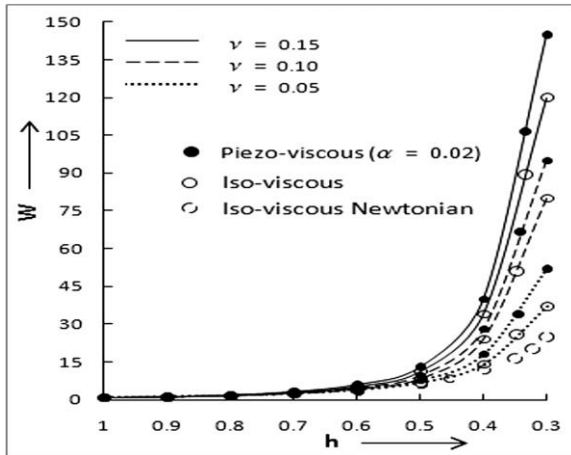


Figure 4. Variation of dimensionless load capacity with the dimensionless film thickness (h) for iso-viscous and piezo-viscous analysis.

Figure 6 shows the relation between the dimensionless film thickness (h) and the dimensionless response time (t) in iso-viscous and piezo-viscous cases for different values of couple stress parameter ν . It is clearly observed that for each value of film thickness, the response time for iso-viscous couple stress fluids is longer than in the iso-viscous Newtonian

fluids and for the same value of couple stress parameter with piezo-viscous effect, a further increase in the response time is observed. Again, the response time increases with the increase of couple stress parameter and for each value of couple stress parameter, the response time in piezo-viscous case is longer than that in the iso-viscous case. Since, the higher film pressure results in a longer response time, the effects of both the couple stresses and the piezo-viscous dependency increase the response time.

Figure 7 shows the variation of dimensionless response time t to attain a particular dimensionless film thickness $h=0.5$ with respect to the piezo-viscous parameter (α) for different values of couple stress parameter ν . The increase in the response time is observed due to the piezo-viscous effect for Newtonian as well as the Couple stress lubricants. The effect of higher value of piezo-viscous parameter produces relatively longer change in the response time for larger values of couple stress parameter. This behaviour is due to the higher film pressure generated by the higher value of couple stress. Again, the effect of couple stresses increase the response time significantly in comparison with the Newtonian.

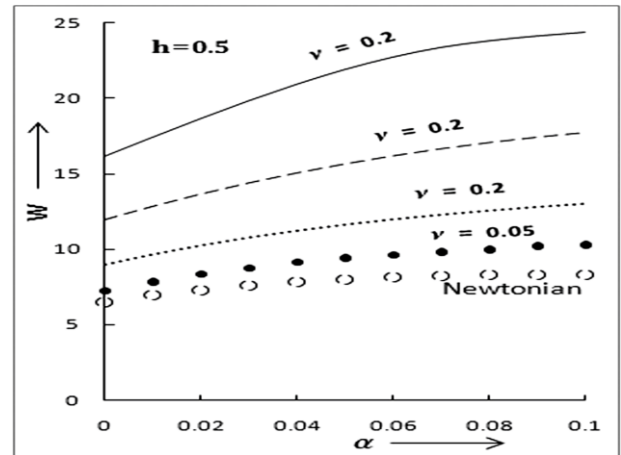


Figure 5. Variation of dimensionless load capacity with piezo-viscous parameter α for different values of couple stress parameter η .

4. Conclusions

Based on Stokes micro-continuum theory of couple stress fluids and the viscosity-pressure dependence, the analytical solution for pressure

distribution, load capacity and response time is obtained using Homotopy Perturbation Method. In the present theoretical analysis, following results have been drawn-

The effect of piezo-viscous dependency as well as the couple stresses enhances the pressure and load carrying capacity significantly.

The effects of both the couple stresses and piezo-viscous dependency increase the response time.

A small value of piezo-viscous parameter ($\alpha = 0.02$) increases the pressure and load capacity by nearly 15% and increase the response time approximately 12% in comparison with the iso-viscous case.

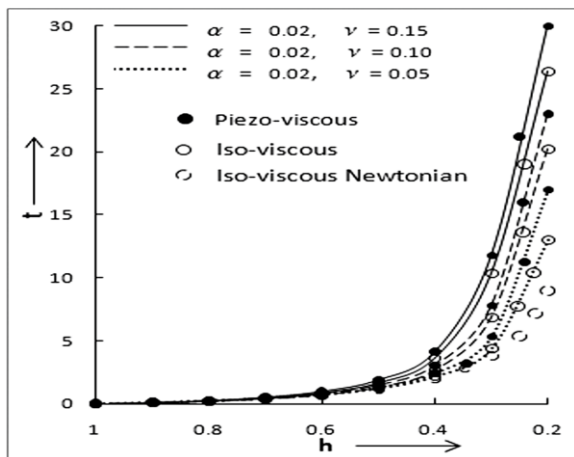


Figure 6. Variation of dimensionless response time with the dimensionless film thickness (h) for iso-viscous and piezo-viscous analysis.

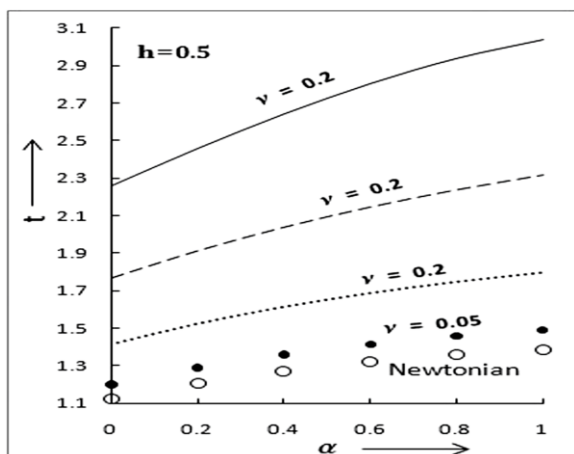


Figure 7. Variation of dimensionless response time with respect to α for different values of η .

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