

Tables of Moments of Sample Extremes of Order Statistics from Discrete Uniform Distribution

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(Received: 11.07.2012; Accepted: 05.08.2013)

Abstract

In this paper, moments of sample extremes of order statistics from discrete uniform distribution are given. For n up to 15, algebraic expressions for the expected values and variances of sample extremes of order statistics from discrete uniform distribution are obtained. It is shown that with the help of the sum $s_n(k)$, one can obtain all moments for sample extremes of order statistics from a discrete uniform distribution. Furthermore, for sample size $k = 20$ and $n = 1(1)20$, numerical results calculated by using Matlab.

Keywords: Order statistics, discrete uniform distribution, moments, sample extremes.

Kesikli Düzgün Dağılımlı Sıra İstatistiklerin Örnek Ekstremlerinin Momentlerinin Tabloları

Özet

Makalede, kesikli düzgün dağılımdaki sıra istatistiklerin örnek ekstremlerinin momentleri verilmiştir. Kesikli düzgün dağılımdaki sıra istatistiklerin örnek ekstremlerinin beklenen değer ve varyansları için $n=15$ ' e kadar cebirsel ifadeler bulunmuştur. $s_n(k)$ toplamı yardımıyla kesikli düzgün dağılımdaki sıra istatistiklerin örnek ekstremlerinin bütün momentlerinin bulunabileceği görülmüştür. Ayrıca, Matlab kullanılarak $k = 20$ ve $n = 1(1)20$ örnek boyutu için sayısal sonuçlar hesaplanmıştır.

Anahtar Kelimeler: Sıra İstatistikleri, kesikli düzgün dağılım, momentler, örnek ekstremleri.

1. Introduction

Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a discrete distributions with probability mass function (pmf) $f(x)$ ($x=0,1,\dots$) and cumulative distribution function (cdf) $F(x)$. Let $X_{1:n} \leq X_{2:n} \leq \dots \leq X_{n:n}$ be the order statistics obtained from above random sample by arranging the observations in increasing order of magnitude. Let $E(X_{r:n}^{(m)})$ denote by $\mu_{r:n}^{(m)}$ ($1 \leq r \leq n, m \geq 1$). For convenience, $\mu_{r:n}$ for $\mu_{r:n}^{(1)}$ and $\sigma_{r:n}^2$ for variance of $X_{r:n}$ will also be used.

The first two moments of order statistics from discrete distributions were proved by Khatri [10]. Arnold et al [3] obtained the first two moments with a different way which were

already obtained by Khatri [10]. Several recurrence relations and identities available for single and product moments order statistics in a sample size n from an arbitrary continuous distribution were extended for the discrete case by Balakrishnan [4]. All the developments on discrete order statistics lucidly accounts by Nagaraja [12]. The first two moments of sample maximum of order statistics from discrete distributions were obtained by Ahsanullah and Nevzorov [1]. For n up to 15, algebraic expressions for the expected values of the sample maximum of order statistics from discrete uniform distribution were obtained by Çalik and Güngör [6]. Furthermore; m th raw moments of order statistics from discrete distribution were proved by Çalik et al [7].

2. The Distribution of Order Statistics

Let $F_{r:n}(x)$ ($r=1,2,\dots,n$) denote by the *cdf* of $X_{r:n}$. Then can be seen easily,

$$F_{r:n}(x) = \Pr\{X_{r:n} \leq x\} = \sum_{i=r}^n \binom{n}{i} [F(x)]^i [1-F(x)]^{n-i}, \quad -\infty < x < \infty \quad (2.1)$$

Thus, we find that the *cdf* of $X_{r:n}$ ($1 \leq r \leq n$) is simply the tail probability of a binomial distribution with $F(x)$ success and n as the number of trials.

The cumulative distribution function of the smallest and largest order statistics follow from (2.1) (when $r=1$ and $r=n$) to be

$$F_{1:n}(x) = 1 - [1 - F(x)]^n, \quad -\infty < x < \infty$$

and

$$F_{n:n}(x) = [F(x)]^n \quad -\infty < x < \infty$$

respectively. Furthermore, by using the identity that

$$\sum_{i=r}^n \binom{n}{i} p^i (1-p)^{n-i} = \int_0^p \frac{n!}{(r-1)!(n-r)!} t^{r-1} (1-t)^{n-r} dt, \quad 0 < p < 1. \quad (2.2)$$

we can write the *cdf* of $X_{r:n}$ from (2.1) equivalently as

$$F_{r:n}(x) = \frac{n!}{(r-1)!(n-r)!} \int_0^{F(x)} t^{r-1} (1-t)^{n-r} dt \quad -\infty < x < \infty \quad (2.3)$$

Observe that all the expressions given above hold for any arbitrary population whether continuous or discrete. For discrete population, the probability mass function of $X_{r:n}$ ($1 \leq i \leq n$) may be obtained from (2.3) by differencing as

$$\begin{aligned} f_{r:n}(x) &= \Pr\{X_{r:n} = x\} = F_{r:n}(x) - F_{r:n}(x-) \\ &= C(r:n) \int_{F(x-)}^{F(x)} t^{r-1} (1-t)^{n-r} dt \end{aligned} \quad (2.4)$$

$$\text{where } C(r:n) = \frac{n!}{(r-1)!(n-r)!} \quad (2.5)$$

Balakrishnan and Rao [5]. In particular, we have

$$\begin{aligned} f_{1:n}(x) &= \int_{F(x-)}^{F(x)} n(1-t)^{n-1} dt \\ &= [\bar{F}(x-)]^n - [\bar{F}(x)]^n \end{aligned} \quad (2.6)$$

$$\bar{F}(x) = 1 - F(x).$$

and

$$f_{n:n}(x) = \int_{F(x-)}^{F(x)} nt^{n-1} dt = [F(x)]^n - [F(x-)]^n. \quad (2.7)$$

3. The Moments of Discrete Order Statistics

Theorem 3.1.

Let $X_1, X_2, \dots, X_n$ be a sample which has F continuous distribution function and $X_{1:n} \leq X_{2:n} \leq \dots \leq X_{n:n}$ indicate order statistics of this sample.

$$U_{1:n} = F(X_{1:n}), \quad U_{2:n} = F(X_{2:n}), \quad \dots, \quad U_{n:n} = F(X_{n:n}),$$

and $U_{1:n}, U_{2:n}, \dots, U_{n:n}$ are order statistics of sample which takes from uniform distribution from interval (0,1) [8].

As pointed out in Theorem 3.1, we can use the transformation, $X_{r:n} = F^{-1}(U_{r:n})$ to obtain the moments of $X_{r:n}$. For example, we can express the means of $X_{r:n}$ as

$$\mu_{r:n} = C(r:n) \int_0^1 F^{-1}(u) u^{r-1} (1-u)^{n-r} du \quad (3.1)$$

where $C(r:n)$ is given by (2.5). However, since $F^{-1}(u)$ does not have a nice form for most of the discrete (as well as absolutely continuous) distributions, this approach is often impractical. When the support B is a subset of nonnegative integers which is the case with several standard discrete distributions, one can use the *cdf* $F_{r:n}(x)$ directly to obtain the m th raw moments of $X_{r:n}$.

Theorem 3.2.

Let B , the support of the distribution, be a subset of nonnegative integers. Then

$$\mu_{r:n}^{(m)} = \sum_{x=0}^{\infty} [(x+1)^m - x^m] (1 - F_{r:n}(x)) \quad (3.2)$$

whenever the moment on the left- hand side is assumed to exist Çalik et al. [7].

In particularly, we also have

$$\mu_{r:n}^{(1)} = \sum_{x=0}^{\infty} [1 - F_{r:n}(x)]$$

and

$$\mu_{r:n}^{(2)} = 2 \sum_{x=0}^{\infty} x [1 - F_{r:n}(x)] + \mu_{r:n}$$

The first two moments of order statistics from discrete distributions were obtained by Khatri [10] and Arnold et al. [3].

In general, these moments are not easy to evaluate analytically. Sometimes, the moments of sample extremes are tractable. Let us see what happens in the case of discrete uniform distribution.

4. The Order Statistics from Discrete Uniform Distribution

Let the population random variable X be discrete uniform with support $B = \{1, 2, \dots, k\}$. Then X is discrete uniform $[1, k]$. Note that its pmf is given by $f(x) = \frac{1}{k}$ and its cdf is $F(x) = \frac{x}{k}$, for $x \in B$. Consequently the cdf of r th order statistics is given by

$$F_{r:n}(x) = \sum_{i=r}^n \binom{n}{i} \left(\frac{x}{k}\right)^i \left(1 - \frac{x}{k}\right)^{n-i}.$$

It can be used directly on tables for cdf of binomial distribution with a selection of x and k . For example, $k=10$ can be expressed as $x=10p$, $p=0, 1(0,1)(1,0)$ for every $x \in B$. Thus

$$F_{r:n}(x) = \sum_{i=r}^n \binom{n}{i} p^i (1-p)^{n-i}, x \in B.$$

can learned from binomial tables and $f_{r:n}(x)$ is obtained by using (2.4).

5. Moments of Order Statistics from Discrete Uniform Distribution

Let $X_1, X_2, \dots, X_n$ be independent and identically distributed uniform random variables from discrete population with pmf $f(x) = \frac{1}{k}$, cdf

$$F(x) = \frac{x}{k}, x = 1, \dots, k. \text{ Then, from (2.4), pmf of } X_{r:n} \text{ can be written}$$

$$\begin{aligned} f_{r:n}(x) &= \int_{F(x-)}^{F(x)} C(r:n) u^{r-1} (1-u)^{n-r} du \\ &= \int_{(x-)/k}^{x/k} C(r:n) u^{r-1} (1-u)^{n-r} du \end{aligned}$$

In particularly, we also have

$$f_{1:n}(x) = \left(\frac{k+1-x}{k}\right)^n - \left(\frac{k-x}{k}\right)^n, x = 1, 2, \dots, k \quad (5.1)$$

and

$$f_{n:n}(x) = \left(\frac{x}{k}\right)^n - \left(\frac{x-1}{k}\right)^n, x = 1, 2, \dots, k. \quad (5.2)$$

Thus, first two moments of $X_{1:n}$

$$E(X_{1:n}) = \sum_{i=1}^k \left(\frac{k+1-i}{k}\right)^n \quad (5.3)$$

and

$$E(X_{1:n}^2) = \sum_{i=1}^k (2i-1) \left(\frac{k+1-i}{k}\right)^n \quad (5.4)$$

respectively. Similarly, first two moments of $X_{n:n}$

$$E(X_{n:n}) = \sum_{i=1}^{k-1} (-1) \left(\frac{i}{k}\right)^n + k \quad (5.5)$$

and

$$E(X_{n:n}^2) = \sum_{i=1}^{k-1} (-2i-1) \left(\frac{i}{k}\right)^n + k^2 \quad (5.6)$$

respectively. Furthermore, from (5.3) and (5.4), variance of sample minimum

$$\sigma_{1:n}^2 = \sum_{i=1}^k (2i-1) \left(\frac{k+1-i}{k}\right)^n - \left[\sum_{i=1}^k \left(\frac{k+1-i}{k}\right)^n\right]^2 \quad (5.7)$$

and similarly, from (5.5) and (5.6) variance of sample maximum

$$\sigma_{n:n}^2 = \sum_{i=1}^{k-1} (-2i-1) \left(\frac{i}{k}\right)^n + k^2 - \left[\sum_{i=1}^{k-1} (-1) \left(\frac{i}{k}\right)^n + k\right]^2 \quad (5.8)$$

for left- hand side summations of (5.3), (5.5), (5.7) and (5.8) can be counted, it can be use following summation

$$s_n(k) = 1^n + 2^n + \dots + k^n = \sum_{i=1}^k i^n \quad (5.9)$$

Zwillinger, D. [13] is obtained algebraic expressions n up to 10 of this summation. By using equality (5.9), algebraic expression of expected values and variances of sample extremes of order statistics from discrete uniform distribution are given Table 1, Table 2 and Table 3. Also, numerical results of these algebraic expressions are given Table 4, using Matlab Program and R.

Table 1. Algebraic expressions for the expected value of sample minimum of order statistics from discrete uniform distribution.

n	μ_{1n}
1	$(1/2)(k+1)$
2	$(1/6)k^{-1}(2k^2+3k+1)$
3	$(1/4)k^{-1}(k^2+2k+1)$
4	$(1/30)k^{-3}(6k^4+15k^3+10k^2-1)$
5	$(1/12)k^{-3}(2k^4+6k^3+5k^2-1)$
6	$(1/42)k^{-5}(6k^6+21k^5+21k^4-7k^2+1)$
7	$(1/24)k^{-5}(3k^6+12k^5+14k^4-7k^2+2)$
8	$(1/90)k^{-7}(10k^8+45k^7+60k^6-42k^4+20k^2-3)$
9	$(1/20)k^{-7}(2k^8+10k^7+15k^6-14k^4+10k^2-3)$
10	$(1/66)k^{-9}(6k^{10}+33k^9+55k^8-66k^6+66k^4-33k^2+5)$
11	$(1/24)k^{-9}(2k^{10}+12k^9+22k^8-44k^6+44k^4-33k^2+10)$
12	$(1/2730)k^{-11}(210k^{12}+1365k^{11}+2730k^{10}-5005k^8+8580k^6-9009k^4+10078k^2-691)$
13	$(1/420)k^{-11}(30k^{12}+210k^{11}+455k^{10}-1001k^8+2145k^6-3003k^4+2275k^2-691)$
14	$(1/90)k^{-13}(6k^{14}+45k^{13}+105k^{12}-273k^{10}+715k^8-1543k^6+1365k^4-691k^2+105)$
15	$(1/48)k^{-13}(3k^{14}+24k^{13}+60k^{12}-182k^{10}+572k^8-1287k^6+1820k^4)-1382k^2+420$

Table 2. Algebraic expressions for the expected value of sample maximum of order statistics from discrete uniform distribution

n	μ_{nn}
1	$(1/2)(k+1)$
2	$(1/6)k^{-1}(4k^2+3k-1)$
3	$(1/4)k^{-1}(3k^2+2k+1)$
4	$(1/30)k^{-3}(24k^4+15k^3-10k^2+1)$
5	$(1/12)k^{-3}(10k^4+6k^3-5k^2+1)$
6	$(1/42)k^{-5}(36k^6+21k^5-21k^4+7k^2-1)$
7	$(1/24)k^{-5}(21k^6+12k^5-14k^4+7k^2-2)$
8	$(1/90)k^{-7}(80k^8+45k^7-60k^6+42k^4-20k^2+3)$
9	$(1/20)k^{-7}(18k^8+10k^7-15k^6+14k^4-10k^2+3)$
10	$(1/66)k^{-9}(6k^{10}+33k^9-55k^8+66k^6-66k^4+33k^2-5)$
11	$(1/24)k^{-9}(22k^{10}+12k^9-22k^8+66k^6-44k^4+33k^2-10)$
12	$(1/2730)k^{-11}(2520k^{12}+1365k^{11}-2730k^{10}+5005k^8-858k^6+9009k^4-4550k^2+691)$
13	$(1/420)k^{-11}(390k^{12}+210k^{11}-455k^{10}+1001k^8-2145k^6+3003k^4-2275k^2+691)$
14	$(1/90)k^{-13}(84k^{14}+45k^{13}-105k^{12}+273k^{10}-715k^8+1287k^6-1365k^4+691k^2-105)$
15	$(1/48)k^{-13}(45k^{14}+24k^{13}-60k^{12}+182k^{10}-572k^8+1287k^6-1820k^4+1382k^2-420)$

Table 3. Algebraic expressions for the variance of sample minimum and maximum of order statistics from discrete uniform distribution

n	$\sigma_{ln}^2 = \sigma_{nn}^2$
1	$(1/12)(k^2 - 1)$
2	$(1/36)k^{-2}(2k^2 + 1)(k^2 - 1)$
3	$(1/240)k^{-2}(9k^2 - 1)(k^2 - 1)$
4	$(1/900)k^{-6}(24k^6 - 21k^4 - 19k^2 + 1)(k^2 - 1)$
5	$(1/1008)k^{-6}(20k^6 - 36k^4 - 15k^2 + 7)(k^2 - 1)$
6	$(1/1764)k^{-10}(27k^{10} - 78k^8 + 6k^6 + 78k^4 - 13k^2 + 1)(k^2 - 1)$
7	$(1/2880)k^{-10}(35k^{10} - 145k^8 + 93k^6 + 213k^4 - 120k^2 + 20)(k^2 - 1)$
8	$(1/8100)k^{-14}(80k^{14} - 445k^{12} + 575k^{10} + 715k^8 - 1499k^6 + 541k^4 - 111k^2 + 9)(k^2 - 1)$
9	$(1/13200)k^{-14}(108k^{14} - 772k^{12} + 1571k^{10} + 911k^8 - 5821k^6 + 4389k^4 - 1683k^2 + 297)(k^2 - 1)$
10	$(1/4356)k^{-18}(30k^{18} - 267k^{16} + 767k^{14} - 25k^{12} - 3622k^{10} + 5690k^8 - 3572k^6 + 1444k^4 - 305k^2 + 25)(k^2 - 1)$
11	$(1/262080)k^{-18}(1540k^{18} - 16660k^{16} + 63420k^{14} - 42400k^{12} - 349707k^{10} + 958873k^8 - 980525k^6) + 641095k^4 - 254800k^2 + 45500)(k^2 - 1)$
12	$(1/7452900)k^{-22}(37800k^{22} - 487725k^{20} + 2359665k^{18} - 3195885k^{16} - 13921600k^{14} + 63345590k^{12} - 104252950k^{10} + 99659850k^8 - 66497141k^6 + 27342319k^4 - 5810619k^2 + 477481)(k^2 - 1)$
13	$(1/176400)k^{-22}(780k^{22} - 11820k^{20} + 70535k^{18} - 148255k^{16} - 399506k^{14} + 3051214k^{12} - 7462327k^{10} + 10192303k^8 - 9968838k^6 + 6659202k^4 - 22666569k^2 + 477481)(k^2 - 1)$
14	$(1/16200)k^{-26}(63k^{26} - 110k^{24} + 7965k^{22} - 23235k^{20} - 36300k^{18} + 515160k^{16} - 1785320k^{14} + 3428080k^{12} - 4587230k^{10} + 4530710k^8 - 3053308k^6 + 1260092k^4 - 268170k^2 + 22050)(k^2 - 1)$
15	$(1/195840)k^{-26}(675k^{26} - 13605k^{24} + 115935k^{22} - 440985k^{20} - 266395k^{18} + 10536085k^{16} + 130345829k^{12} - 231687560k^{10} + 313890720k^8 - 310871860k^6 + 208610740k^4 - 83680800k^2 + 14994000)(k^2 - 1)$

Table 4. Expected values and variances of sample extremes of order statistics from discrete uniform distribution

k	n	μ_{ln}	σ_{ln}^2	k	n	μ_{nn}	σ_{nn}^2
20	1	10.5000	33.2500	20	1	10.5000	33.2500
	2	7.1750	22.1944		2	13.8250	22.1944
	3	5.5125	14.9583		3	15.4875	14.9583
	4	4.5167	10.6167		4	16.4833	10.6167
	5	3.8542	7.8810		5	17.1458	7.8810
	6	3.3821	6.0630		6	17.6179	6.0630
	7	3.0291	4.7988		7	17.9709	4.7988
	8	2.7555	3.8861		8	18.2445	3.8861
	9	2.5374	3.2065		9	18.4626	3.2065
	10	2.3597	2.6872		10	18.6403	2.6872
	11	2.2123	2.2817		11	18.7877	2.2817
	12	2.0882	1.9592		12	18.9118	1.9592
	13	1.9824	1.6984		13	19.0176	1.6984
	14	1.8913	1.4847		14	19.1087	1.4847
	15	1.8120	1.3074		15	19.1880	1.3074
	16	1.7426	1.1587		16	19.2574	1.1587
	17	1.6812	1.0327		17	19.3188	1.0327
	18	1.6268	0.9252		18	19.3732	0.9252
	19	1.5782	0.8326		19	19.4218	0.8326
	20	1.5345	0.7523		20	19.4655	0.7523

6. Discussion and Conclusion

The current study presents the obtained algebraic expression of the expected values and variances of the sample extremes of order statistics from discrete uniform distribution, as shown in Tables 5.1, 5.2 and 5.3. Using the obtained algebraic expressions, these expected values and variances are computed. As shown in Table 5.4, different values can be obtained for k and n .

Moments of order statistics are of great importance in many statistical problems. The information obtained about the means, variances and covariances of moment of order statistics enables the evaluation of the expected values and variances of the linear functions of order statistics. Obtained algebraic and numerical results for order statistics are applicated others department. In a study entitled "Natural selection and veridical perceptions", Mark et al. [11] used the expected values of the sample maximum of order statistics from discrete uniform distribution.

During the last decades, computer technology has developed considerably in relation to statistical analyses and computations. Furthermore, software programs such as artificial neural networks, several algorithms, etc. have performed impressively in carrying out statistical problems. Evans et al. [9] presented an algorithm for computing the probability density function of order statistics drawn from discrete parent populations and used exact bootstrapping analysis, which illustrates the utility of the presented algorithm. Computer-aided algorithms give good results on the computations related to order statistics. Adatia [2] derived an explicit expression for the expected value of the product of two order statistics from the geometric distribution and discussed a method of computation for the expected values and covariances of order statistics. Other studies have focussed on computer-aided computations or algorithms generated by some software programs. In parallel with the developments in computer-based technology, in the next phase of the study, we want to create a program which computes the means and variances of the sample extremes of order statistics for the discrete distributions.

In conclusion, using the obtained equality previously described, all the moments for the sample extremes of order statistics from the discrete uniform distribution can be achieved. It is recommended that (2.4) equality be applied to other discrete distributions. Further studies may focus on a software program which computes the means and variances of the sample extremes of order statistics from any discrete distributions.

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ADD 1.

Matlab software for the expected value of sample minimum at Table 5.4

```
top=0;
for n=1:10
for i=1:k
top=top+((k+1-i)/k)^n;
end
a(n)=top;
top=0;
end
disp(a)
```

ADD 2.

Matlab software for the variance of sample minimum at Table 5.4

```
k=input('k y1 gir');
top=0; top2=0;
for n=1:10
for i=1:k
top=top+(2*i-1)*(((k+1-i)/k)^n);
top2=top2+(((k+1-i)/k)^n);
end
top2=(top2)^2; b(n)=top-top2;
top=0; top2=0;
end
disp(b)
```

ADD 3.

Matlab software for the expected value of sample maximum at Table 5.4

```
k=input('k y1 gir');
top=0;
for n=1:10
for i=1:(k-1)
top=top+((-1)*((i/k)^n));
end
a(n)=k+top;
top=0;
end
disp(a)
```

ADD 4.

Matlab software for the variance of sample maximum at Table 5.4

```
k=input('k y1 gir');
top=0; top2=0;
for n=1:10
for i=1:k-1
top=top+(-2*i-1)*(i/k)^n;
top2=top2+((-1)*((i/k)^n));
end
top=top+k^2; top2=top2+k; b(n)=top-top2^2;
top=0; top2=0;
end
disp(b)
```